Math 210
Exam 1 - Version 1 **Name:**______________________________

Instructions: You will have 55 minutes to complete this exam. The credit given on each problem will be proportional to the amount of correct work shown. Answers without supporting work will receive little credit.

1. (10 points) Use a truth table to determine whether or not the proposition $[(p \wedge q) \to r] \to \neg q$ is a tautology.

p	q	r	$p \wedge q$	$\neg q$	$(p \wedge q) \to r$	$[(p \wedge q) \to \neg r] \to \neg q$
T	T	T	T	F	T	F
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	F	T	F
F	T	F	F	F	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Since the final column of the truth table above has some 'F' entries, we see that the given statement is not a tautology.

2. (12 points) Use truth tables to determine whether or not the following statements are logically equivalent: $p \to (q \to r)$ and $(p \to q) \to r$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \to q$	$(p \to q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	F	T	T	F
F	F	T	T	T	T	T
F	F	F	T	T	T	F

Since the truth table columns corresponding to these two statements to not match, we see that the two given statements are not logically equivalent.

3. (5 points each) Translate each of the following into symbolic form (use propositional logic). Be sure to clearly define all variables used.

   (a) I am tired but I am not hungry.

   Let t represent: I am tired. Let h represent: I am hungry.

   The most natural symbolic translation of the statement is: $t \wedge \neg h$.

   (b) I get a sunburn whenever I go to the beach.

   Let s represent: I get a sunburn. Let b represent: I go to the beach.

   The most natural symbolic translation of the statement is: $b \to s$.

   (c) Having a valid I.D. is sufficient for getting into the club.

   Let V represent: having a valid I.D. Let C represent: getting into the club.

   The most natural symbolic translation of the statement is: $V \to C$.

4. (5 points each) Let x and y be integers. Determine the truth value of each of the following. You must justify your answer to receive full credit.

   (a) $\forall x \exists y (x + y = 0)$

   TRUE

   To see this, note that, given an integer x, if we let $y = -x$, then y is also an integer, and $x + y = x + (-x) = 0$.

   (b) $\exists y \forall x (xy = 0)$

   TRUE

   To see this, note that we may take our choice of y to be zero (since y is quantified first, in order for this statement to be true, we must be able to provide an example of a specfic y value that works for **any** x value).

   If $y = 0$, then for any x, $xy = x(0) = 0$.

   (c) $\forall x \exists y (xy = 1)$

   FALSE

   To see this, note that unlike in part (b), given the order of quantification, we must choose x first and then show that for any possible choice of value for x, there is a correcponding value for y so that $xy = 1$.

   Here is a specific counterexample: choose $x = 2$. Then in order to have $xy = 1$, we need $y = \frac{1}{2}$. However, note that the domain definition above states that y is an integer. Then the only possible value for y lies outside the stated domain. Then no integer y exists so that $xy = 1$.

5. (6 points each) Let $A(x, y)$ denote the predicate "x is afraid of y", let $F(x)$ denote the predicate "x has a flashlight." and let $C(x)$ denote "x went to the creepy carnival". Where the domain of x is the set of all people and y is the set of all things.

   (a) Translate the following symbolic statement into English: $\forall x\,(A(x, \text{Clowns}) \rightarrow \neg C(x))$

   One translation is as follows: "Everyone who is afraid of clowns did not go to the creepy carnival."

   Note: Your score on translation exercises is based on the overall correctness of your translation (how accurately it reflects the meaning of the original symbolic statement), not on the specific phrasing used. For this part, you lost points if your translation insinuated that everyone is afraid of clowns, which is more than what this statement claims.

   (b) Translate the following symbolic statement into English: $\forall x \exists y(A(x, y))$

   One translation is as follows: "Everyone is afraid of something."

   (c) Write the following statement in symbolic form: Everyone who has a flashlight is not afraid of the dark.

   $\forall x\,(F(x) \rightarrow \neg A(x, \text{the dark}))$

   (d) Write the following statement in symbolic form: There is exactly one person who is afraid of clowns but still went to the creepy carnival.

   $\exists! x\,(A(x, \text{Clowns}) \wedge C(x))$

6. (5 points each) Write the simplified negation of each of the following statements in symbolic form.

   (a) $\forall x\,(P(x) \wedge R(x))$

   Negating this gives: $\exists x \neg\,(P(x) \wedge R(x)) \equiv \exists x\,(\neg P(x) \vee \neg R(x))$

   (b) $\exists y \forall x\,(Q(x, y) \rightarrow T(x, y))$

   Negating this gives: $\forall y \exists x \neg\,(Q(x, y) \rightarrow T(x, y)) \equiv \forall y \exists x\,(Q(x, y) \wedge \neg T(x, y))$

7. (7 points) Write the simplified negation of the following statement in plain English (it may be helpful to negate the statement symbolically first).

   Everyone who has been to California has been to Disneyland.

   Symbolically translating this expression is not absolutely necessary, but is helpful. One possible translation is:

   $\forall x\,(C(x) \to D(x))$, where $C(x)$ is "x has been to California" and $D(x)$ is "x has been to the Disneyland", and x is the set of all people.

   Negating this gives: $\exists x \neg\,(C(x) \to D(x)) \equiv \exists x\,(C(x) \wedge \neg D(x))$

   This translates as: "There is at least one person who has been to California but has not been to Disneyland."

8. (12 points) Use a 2-column proof to show that the following statement is a tautology: $\neg\,(p \to q) \to p$

   Note that there is more then one correct proof that could be given depending on the order of steps chosen. Here is one possibility:

Statement	**Reason**
(1) $\neg\,(p \to q) \to p$	Given
(2) $(p \wedge \neg q) \to p$	Negation of a Conditional (applied to inner conditional)
(3) $\neg\,(p \wedge \neg q) \vee p$	Conditional to Disjunction
(4) $(\neg p \vee \neg(\neg q)) \vee p$	De Morgan's Law
(5) $(\neg p \vee q) \vee p$	Double Negation
(6) $\neg p \vee (q \vee p)$	Associative Law
(7) $\neg p \vee (p \vee q)$	Commutative Law
(8) $(\neg p \vee p) \vee q$	Associative Law
(9) $T \vee q$	Negation Law
(10) T	Domination Law

   Hence this statement is a tautology.