

1. Translate the following English statements into symbolic form (use propositional sentences).
 - (a) I will not pass this class unless I go to class every day and do all of the homework exercises.
 - (b) I lock the doors and close the windows whenever I leave to go to work.
 - (c) Getting up on time and getting ready quickly is sufficient for arriving at work on time.
 - (d) Practicing an hour a day and getting private lessons twice a week is necessary for playing in the wind ensemble.
2. Use truth tables to determine whether or not the following pairs of statements are logically equivalent.
 - (a) $[(p \wedge q) \rightarrow r]$ and $(p \rightarrow r) \wedge (q \rightarrow r)$
 - (b) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$
 - (c) $p \wedge (q \vee \neg r)$ and $(p \wedge q) \vee \neg(\neg p \vee r)$
3. Use truth tables to determine which of the following statements are tautologies:
 - (a) $(p \oplus q) \leftrightarrow [(p \vee q) \wedge \neg(p \wedge q)]$
 - (b) $(p \rightarrow q) \leftrightarrow [\neg(p \wedge \neg q)]$
 - (c) $p \rightarrow (\neg q \vee r \vee \neg r)$
4. Use a 2-column proof to show that the following statements are tautologies:
 - (a) $[\neg p \wedge (p \vee q)] \rightarrow q$
 - (b) $\neg p \rightarrow (p \rightarrow q)$
5. Use a 2-column proof to show that each pair of logical statements are logically equivalent.
 - (a) $(r \vee p) \rightarrow (r \vee q)$ and $r \vee (p \rightarrow q)$.
 - (b) $p \rightarrow q$ and $[(p \wedge \neg q) \rightarrow \neg p]$
6. Determine whether or not the following statements are satisfiable.
 - (a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee r)$
 - (b) $(p \vee q \vee \neg r \vee s) \wedge (p \vee \neg q \vee r \vee \neg s) \wedge (\neg p \vee \neg q \vee r \vee \neg s)$
7. Suppose that there is a certain town in which all the men either shave themselves, or they are shaved by the town barber Figaro (who is male). Suppose Vinny is too cheap to have the barber shave him. Let $S(x, y)$ be the two variable predicate "person x is shaved by person " y ". First translate the given statement into English, then determine the truth value of the statement and justify your answer.
 - (a) $\forall x (S(x, \text{Figaro}) \vee S(x, x))$
 - (b) $\forall x (\neg S(x, x) \rightarrow S(x, \text{Figaro}))$
 - (c) $\forall x (S(x, \text{Figaro}) \rightarrow \neg S(x, x))$
 - (d) $\exists y \forall x (S(x, y))$
 - (e) $\exists! x (S(x, x) \wedge S(x, \text{Figaro}))$
8. Translate each of the following statements into quantified predicate form. Make sure to define each predicate used and state the domain of each variable.
 - (a) At least one person in this neighborhood watches television on Monday but not on Wednesday.
 - (b) There is a person who has run a marathon in every state in the United States of America.

9. Write the negation of each of the following statements (First write each statement symbolically, then negate the symbolic statement, and finally, translate the negation back into plain English).

(a) Everyone who took their driver's exam today passed the exam.

(b) Some people like bowling and tennis.

(c) If everyone passed the exam then everyone studied for the exam.

10. Negate each of the statements (your answer should be in symbolic form)

(a) $\forall x \forall y [(x > 0) \wedge (y < 0)] \rightarrow (xy \geq 0)$

(b) $\exists x \forall y \forall z [(F(x, y) \wedge G(x, z)) \rightarrow H(y, z)]$

(c) $\forall x \exists! y [S(x, y) \vee \neg R(x, y)]$

11. Let x , y and z be integers. Determine the truth value of each of the following.

(a) $\forall x \exists y (x + y = 1)$

(b) $\forall x \forall y \exists z (xy < z)$

(c) $\exists z \forall x \forall y (xy < z)$