Math 210
Exam 2 - Practice Problems

1. For each of the following, determine whether the statement is True or False.

(a) $\emptyset \subseteq \{a,b,c,d\}$ TRUE
(b) $\emptyset \in \{a,b,c,d\}$ FALSE
(c) $\emptyset \in \{a,b,\emptyset\}$ TRUE
(d) $\emptyset \subseteq \{a,b,\emptyset\}$ TRUE
(e) $\{a,b\} \subset \{a,b\}$ FALSE
(f) $0 \in \{0,\{1\},\{0,1\}\}$ TRUE
(g) $1 \in \{0,\{1\},\{0,1\}\}$ FALSE
(h) $\{0,1\} \in \{0,\{1\},\{0,1\}\}$ TRUE
(i) $\{0,1\} \subset \{0,\{1\},\{0,1\}\}$ FALSE

2. Given the set $B = \{a, b, \{a, b\}\}$

(a) Find $|B|$.

$|B| = 3$

(b) Find $\mathcal{P}(B)$

$\mathcal{P}(B) = \{\emptyset, \{a\}, \{b\}, \{\{a,b\}\}, \{a,b\}, \{a,\{a,b\}\}, \{b,\{a,b\}\}, \{a,b,\{a,b\}\}\}$

3. Given that $A = \{1,2,3\}$ and $B = \{a,b,c,d,e,f\}$

(a) List the elements in $A \times A$.

$A \times A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

(b) How many elements are in $A \times B$?

$|A \times B| = |A| \cdot |B| = 3 \cdot 6 = 18$.

(c) How many elements are in $A \times (B \times B)$?

First notice that $|B \times B| = |B| \cdot |B| = 6 \cdot 6 = 36$.

Then $|A \times (B \times B)| = |A| \cdot |B \times B| = 3 \cdot 36 = 108$.

4. Find the set of all elements that make the predicate $Q(x) : x^2 < x$ true (where the domain of x is all real numbers).

First notice that if $x > 1$, then $x \cdot x > 1 \cdot x$, so $x^2 > x$.

If $x < 0$, then since $x^2 > 0$ for all real x, $x^2 > 0 > x$.

If $x = 0$, then $0^2 = 0$. Similarly, if $x = 1$, then $1^2 = 1$.

If $0 < x < 1$, then $x \cdot x < 1 \cdot x$, or $x^2 < x$.

Hence the set of all elements that make the predicate $Q(x) : x^2 < x$ true is $A = \{x \,|\, 0 < x < 1\}$.

5. Given that $A = \{0,2,4,6,8,10,12\}$, $B = \{0,2,3,5,7,11,12\}$ and $C = \{1,2,3,4,6,7,8,9\}$ are all subsets of the universal set $U = \{0,1,2,3,4,5,6,7,8,9,10,11,12\}$, find each of the following:

(a) $A - B = \{4,6,8,10\}$

(b) $\overline{A} = \{1,3,5,7,9,11\}$

(c) $A \cap B = \{0,2,12\}$

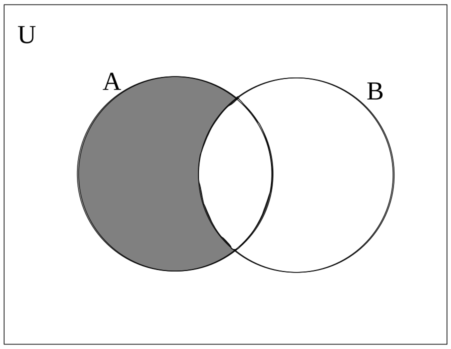
(d) $A \cup (B \cap C)$
$B \cap C = \{2,3,7\}$, so $A \cup (B \cap C) = \{0,2,3,4,6,7,8,10,12\}$

(e) $A - (\overline{B} \oplus C)$
$\overline{B} = \{1,4,6,8,9,10\}$, so $\overline{B} \oplus C = \{2,3,7,10\}$.
Hence $A - (\overline{B} \oplus C) = \{0,4,6,8,12\}$

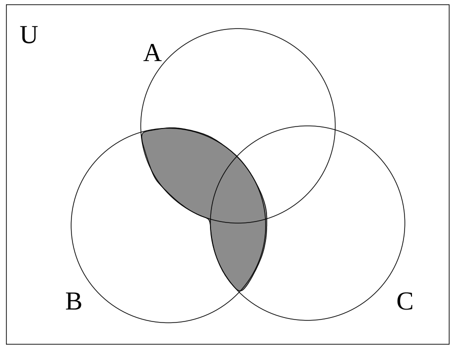
(f) $(A \cap C) \cup (B - \overline{A})$
$A \cap C = \{2,4,6,8\}$, $\overline{A} = \{1,3,5,7,9,11\}$, and so $B - \overline{A} = \{0,2,12\}$.
Thus $(A \cap C) \cup (B - \overline{A}) = \{0,2,4,6,8,12\}$

6. Draw Venn Diagrams representing each of the following sets:

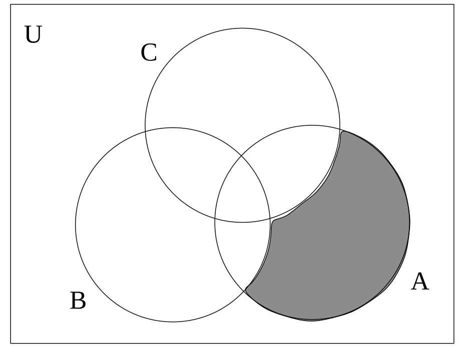
(a) $A - B$



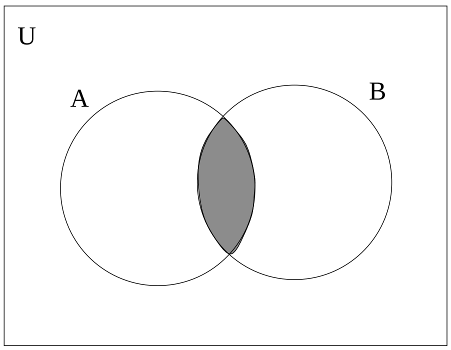
(c) $(A \cup C) \cap B$



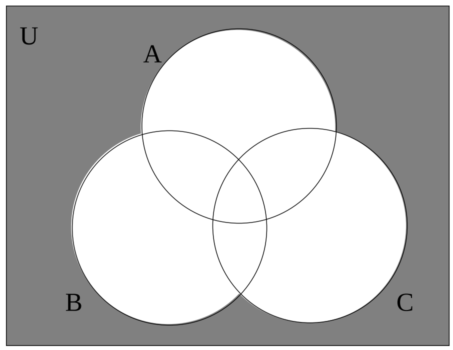
(e) $A - (B \cup C)$



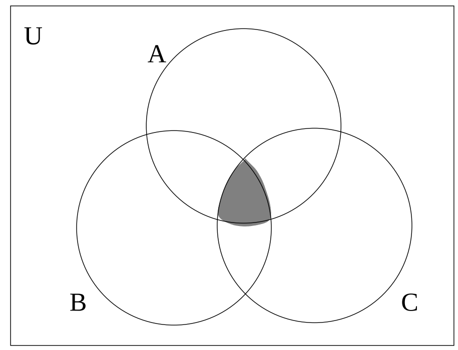
(b) $B - \overline{A}$



(d) $\overline{A \cup B \cup C}$



(f) $(A \cap B) - \overline{C}$



7. Use a membership table to show that $(B - A) \cup (C - A) = (B \cup C) - A$.

A	B	C	$B - A$	$C - A$	$(B - A) \cup (C - A)$
1	1	1	0	0	0
1	1	0	0	0	0
1	0	1	0	0	0
1	0	0	0	0	0
0	1	1	1	1	1
0	1	0	1	0	1
0	0	1	0	1	1
0	0	0	0	0	0

A	B	C	$B \cup C$	$(B \cup C) - A$
1	1	1	1	0
1	1	0	1	0
1	0	1	1	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	0	0

Since the last columns of these membership tables are identical, these two sets are equal.

8. Use a 2-column proof to verify the set identity: $A \cup (A \cap B) = A$.

Statement	**Reason**
$A \cup (A \cap B)$	Given
$= \{x \mid (x \in A) \vee [(x \in A) \wedge (x \in B)]\}$	Definition of union and definition of intersection.
$= \{x \mid x \in A\}$	Absorption Law for logical statements.
$= A$	Definition of A

The proof given in the table above verifies that these sets are equal, so this identity is always valid.

9. For each of the following, either prove the statement or show that it is false using a counterexample.

(a) $(A - B) - C = A - (B - C)$

FALSE. Consider the counterexample: $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5\}$, and $C = \{3, 4, 5, 6\}$.
Then $A - B = \{1, 4\}$, so $(A - B) - C = \{1\}$, while $B - C = \{2\}$, so $A - (B - C) = \{1, 3, 4\}$.
Then $(A - B) - C \neq A - (B - C)$

(b) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

We will prove this equality using membership tables:

A	B	C	$B \oplus C$	$A \oplus (B \oplus C)$
1	1	1	0	1
1	1	0	1	0
1	0	1	1	0
1	0	0	0	1
0	1	1	0	0
0	1	0	1	1
0	0	1	1	1
0	0	0	0	0

A	B	C	$A \oplus B$	$(A \oplus B) \oplus C$
1	1	1	0	1
1	1	0	0	0
1	0	1	1	0
1	0	0	1	1
0	1	1	1	0
0	1	0	1	1
0	0	1	0	1
0	0	0	0	0

Since the last columns of these membership tables are identical, these two sets are equal.

(c) $A \cap (B - C) = (A \cap B) - (A \cap C)$

A	B	C	$B - C$	$(A \cap (B - C)$
1	1	1	0	0
1	1	0	1	1
1	0	1	0	0
1	0	0	0	0
0	1	1	0	0
0	1	0	1	0
0	0	1	0	0
0	0	0	0	0

A	B	C	$A \cap B$	$A \cap C$	$(A \cap B) - (A \cap C)$
1	1	1	1	1	0
1	1	0	1	0	1
1	0	1	0	1	0
1	0	0	0	0	0
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

Since the last columns of these membership tables are identical, these two sets are equal.

10. Consider the function $f(x) = |x|$

(a) Suppose that the domain of this function is $\mathbb{R}$ and the co-domain is $\mathbb{R}$. Find the range of f. Is f 1-1? Is f onto? Justify your answers.

Recall that if $x \geq 0$, then $f(x) = x$ and if $x < 0$, then $f(x) = -x$. From this, we see that the range of f is $\{x : x \in \mathbb{R}, x \geq 0\}$.

Notice that $f(1) = f(-1) = 1$, so f is not one-to-one.

Based on the range we found, we see that f is not onto. For example, there is no x that maps to -1.

(b) Suppose that the domain of this function is $\mathbb{N}$ and the co-domain is $\mathbb{N}$. Find the range of f. Is f 1-1? Is f onto? Justify your answers.

If we change the domain and co-domain to $\mathbb{N}$, then f has range $\{x : x \in \mathbb{N}\} = \mathbb{N}$. Thus f is onto.

With the given domain and co-domain, f is also one-to-one, since on this domain, $f(x) = x$ for all $x \in \mathbb{N}$.

(c) Suppose $S = \{-2, -1, 0, 1, 2\}$. Find $f(S)$ (the image of the set S under f). Find $f^{-1}(S)$ (the preimage of the set S under f).

$f(S) = \{0, 1, 2\}$. However, since -2 and -1 are not legal images, $f^{-1}(S)$ is undefined.

11. For each of the following functions, determine whether f is a one-to-one. Also determine whether f is onto. Justify your answers.

(a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3 - x$

Notice that $f(0) = f(1) = 0$. Therefore, f is not one-to-one. To see that f is onto, notice that $f'(x) = 3x^2 - 1$, so $f'(x) \geq 0$ whenever $|x| \geq \frac{\sqrt{3}}{3}$. From this, we can deduce that f is increasing both on $(\infty, -1]$ and on $[1, \infty)$. Also, note that $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$.

Finally, since $f(1) = 0$ and $f(-1) = 0$, f attains all positive values in $\mathbb{R}$ on the interval $[1, \infty)$, and f attains all negative values in $\mathbb{R}$ on the interval $(-\infty, -1]$. Thus f is onto.

(b) $f : \mathbb{R}^+ \to \mathbb{R}^+$ $f(x) = x^2$

Notice that since we have restricted the domain to include only positive values, if $f(a) = f(b)$, then $a^2 = b^2$, so $a = b$. Hence f is one-to-one.

Also, if we consider $k \in \mathbb{R}^+$ and let $x = \sqrt{k}$ (which is defined since $k \geq 0$). Then $f(x) = (\sqrt{k})^2 = k$.

(c) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ $f(m, n) = m^2 - n$

f is not one-to-one, since $f(1, 1) = f(-1, 1) = f(0, 0) = 0$.

f is onto. To see this, let $k \in \mathbb{N}$. Let $m = 0$, and $n = -k$. Then $f(m, n) = 0 - (-k) = k$.

(d) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ $f(m, n) = m^2 - n^2$

f is not one-to-one, since $f(1, 1) = f(-1, 1) = f(0, 0) = 0$.

f is not onto. To see this, let $k = 6$. Now, the square numbers are $0, 1, 4, 9, 16, 25, 36, ...$

If we subtract consecutive squares, as we proved on a previous homework exercise, we get the odd numbers $1, 3, 5, 7, 9, 11, \cdots$.

To obtain an even difference, we must subtract non-consecutive squares. Notice that $4 - 0 = 4$, $9 - 1 = 8$, and $16 - 4 = 12$. Since our previous result shows that the difference between perfect squares increases as their size increases, we can see that there is no way of writing 6 as a difference of two perfect squares. Hence f is not onto.

[Notice that we could prove that the difference between two perfect squares is either odd or is a multiple of 4, but that goes beyond what was asked for in this problem.]

12. Prove or Disprove: Suppose $f : B \to C$ and $g : A \to B$. If f is one-to-one and g is onto, then $f \circ g$ is one to one.

This statement is false. For example, Let $A = B = C = \mathbb{N}$ consider $f(n) = n$ and $g(n) = \lceil \frac{n}{2} \rceil$.

f is the identity map, so f is one-to-one. g is onto since $g(2n) = n$ for all n. However, notice that $(f \circ g)(n) = g(n)$ for all n. Therefore, $f \circ g$ is not one-to-one, since $(f \circ g)(1) = (f \circ g)(2) = 1$.

13. Prove or Disprove: Suppose $f : B \to C$ and $g : A \to B$. If f is one-to-one and g is onto, then $f \circ g$ is onto.

This statement is false. For example, Let $A = B = C = \mathbb{N}$ consider $f(n) = 2n$ and $g(n) = n$.

g is the identity map, so g is one-to-one. f is one-to-one since if $f(a_1) = f(a_2)$, then $2a_1 = 2a_2$, so $a_1 = a_2$. However, notice that $(f \circ g)(n) = 2n$ for all n. Therefore, $f \circ g$ is not onto, since there is no input n such that $(f \circ g)(n) = 1$.

14. Find the first five terms of each of the following sequences (start with $n = 1$):

(a) $a_n = 3n - 1$

$a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$

(b) $b_n = (-1)^n n^2$

$b_1 = -1$, $b_2 = 4$, $b_3 = -9$, $b_4 = 16$, $b_5 = -25$

(c) $c_n = n^{n-1}$

$c_1 = 1^0 = 1$, $c_2 = 2^1 = 2$, $c_3 = 3^2 = 9$, $c_4 = 4^3 = 64$, $c_5 = 5^4 = 625$

(d) $a_n = 2a_{n-1}$, $a_0 = 5$

$a_1 = 10$, $a_2 = 20$, $a_3 = 40$, $a_4 = 80$, $a_5 = 160$

(e) $b_n = b_{n-1} + n^2$, $b_0 = 7$

$b_1 = 7 + 1 = 8$, $b_2 = 8 + 4 = 12$, $b_3 = 12 + 9 = 21$, $b_4 = 21 + 16 = 37$, $b_5 = 37 + 25 = 62$

(f) $c_n = c_{n-1} + nc_{n-2}$, $c_0 = 2$, $c_1 = 3$

$c_1 = 3$, $c_2 = 3 + 2(2) = 7$, $c_3 = 7 + 3(3) = 16$, $c_4 = 16 + 4(7) = 44$, $c_5 = 44 + 5(16) = 124$

15. Find three different sequences beginning with the terms $a_1 = 1$, $a_2 = 2$ and $a_3 = 4$.

There are many possible answers to this question. Here are some possibilities:

$a_n = 2^{n-1}$

$b_n = b_{n-1} + b_{n-2} + 1$; $b_1 = 1$; $b_2 = 2$

$c_n = 2c_{n-1}$; $c_1 = 1$

16. Determine whether or not each of the following is a solution to the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$

(a) $a_n = 0$

Notice that if $a_n = 0$ for all n, then $8a_{n-1} - 16a_{n-2} = 8(0) - 16(0) = 0 = a_n$, so this sequence is a solution to the given recurrence relation.

(b) $a_n = 2^n$

Notice that if $a_n = 2^n$ for all n, then $8a_{n-1} - 16a_{n-2} = 8(2^{n-1}) - 16(2^{n-2}) = 4{\cdot}2{\cdot}2^{n-1} - 4{\cdot}4{\cdot}2^{n-2} = 4{\cdot}2^n - 4{\cdot}2^n = 0 \neq a_n$, so this sequence is **not** a solution to the given recurrence relation.

(c) $a_n = n4^n$

Notice that if $a_n = n4^n$ for all n, then $8a_{n-1} - 16a_{n-2} = 8((n-1)4^{n-1}) - 16((n-2)4^{n-2}) = 2(4)4^n(n-1) - 4^2 4^{n-2}(n-2) = 2(4^n)(n-1) - 4^n(n-2) = 4^n[2(n-1)-(n-2)] = 4^n[2n-2-n+2] = 4^n \cdot n = n4^n$, so this sequence is a solution to the given recurrence relation.

(d) $a_n = 2 \cdot 4^n + 3n \cdot 4^n$

Notice that if $a_n = 2 \cdot 4^n + 3n \cdot 4^n$ for all n, then $8a_{n-1} - 16a_{n-2} = 8(2 \cdot 4^{n-1} + 3(n-1) \cdot 4^{n-1}) - 16((2 \cdot 4^{n-2} + 3(n-2) \cdot 4^{n-2}) = 8 \cdot 4^{n-1}[2 + 3(n-1)] - 16 \cdot 4^{n-2}[2 + 3(n-2)] = 2 \cdot 4^n[2 + 3n - 3] - \cdot 4^n[2 + 3n - 6] = 4^n[2(3n-1) - 3n - 4] = 4^n(3n+2) = 2\dot{4}^n + 3n \cdot 4^n$, so this sequence is a solution to the given recurrence relation.

17. Find a recurrence relation satisfying each of the following:

(a) $a_n = 3n - 2$

$a_n = a_{n-1} + 3$; $a_1 = 1$

(b) $a_n = 3^n$

$a_n = 3a_{n-1} + 3$; $a_0 = 1$

(c) $a_n = n^2$

$a_n = a_{n-1} + 2n - 1$; $a_0 = 0$

18. Find the solution to each of the following recurrence relations and initial conditions

(a) $a_n = 4a_{n-1}$, $a_0 = 1$

Notice that $a_n = 4a_{n-1}$ and $a_{n-1} = 4a_{n-2}$. Therefore, $a_n = 4(4a_{n-2}) = 4^2 a_{n-2}$.

Continuing in this fashion, $a_n = 4^2 a_{n-2} = 4^2(4a_{n-3}) = 4^3 a_{n-3} = \cdots = 4^n a_0$ where $a_0 = 1$.

Hence this sequence has explicit form: $a_n = 4^n$.

(b) $a_n = a_{n-1} + 4$, $a_0 = 4$

Notice that $a_n = a_{n-1} + 4$ and $a_{n-1} = a_{n-2} + 4$. Therefore, $a_n = (a_{n-2} + 4) + 4 = a_{n-2} + 2(4)$.

Continuing in this fashion, $a_n = a_{n-2} + 2(4) = (a_{n-3} + 4) + 2(4) = a_{n-3} + 3(4) = \cdots = a_0 + n(4)$ where $a_0 = 4$.

Hence this sequence has explicit form: $a_n = 4n + 4$.

(c) $a_n = a_{n-1} + n$, $a_0 = 1$

Notice that $a_n = a_{n-1} + n$ and $a_{n-1} = a_{n-2} + (n-1)$. Therefore, $a_n = a_{n-2} + (n-1) + n$.

Continuing in this fashion, $a_n = a_{n-3} + (n-2) + (n-1) + n = \cdots = a_0 + \sum_{k=1}^{n} k$ where $a_0 = 1$.

Hence $a_n = 1 + \sum_{k=1}^{n} k = 1 + \frac{n(n+1)}{2} = \frac{n^2+n+2}{2}$

19. Compute the value of each of the following summations:

(a) $\displaystyle\sum_{k=1}^{5} 2k = 2 + 4 + 6 + 8 + 10 = 30$

(b) $\displaystyle\sum_{i=0}^{3} 3^i = 1 + 3 + 9 + 27 = 40$

(c) $\displaystyle\sum_{j=3}^{13} 5 = 11(5) = 55$

(d) $\displaystyle\sum_{j=2}^{5} 2^j - 2j = (4-4) + (8-6) + (16-8) + (32-10) = 0 + 2 + 8 + 22 = 32$

(e) $\displaystyle\sum_{i=1}^{3}\sum_{j=0}^{2} ij^2 = \sum_{i=1}^{3}\left(i(0^2) + i(1^2) + i(2^2)\right) = \sum_{i=1}^{3} 5i = 5(1) + 5(2) + 5(3) = 30$

(f) $\displaystyle\sum_{j=0}^{2}\sum_{i=1}^{2} ij^2 = \sum_{j=0}^{2}\left(j^2 + 2j^2\right) = \sum_{j=0}^{2} 3j^2 = 3(0) + 3(1) + 3(2) = 9$

20. Prove that $n^5 - n$ is divisible by 5 for any non-negative integer n.

We will proceed by induction. **Base Case:** $n = 0$ Notice that $0^5 - 0 = 0$. Since $5 \cdot 0 = 0$, 0 is divisible by 5.

Induction Step: Suppose that $k^5 - k$ is divisible by 5, and consider $(k+1)^5 - (k+1)$.
Expanding this, $(k+1)^5 - (k+1) = (k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1) - (k+1) = k^5 + 5k^4 + 10k^3 + 10k^2 + (5k - k)$
$= 5k^4 + 10k^3 + 10k^2 + 5k + (k^5 - k)$.
Since 5 divides $k^5 - k$, by the induction hypothesis, 5 divides every term of the previous expression.
This 5 divides $(k+1)^5 - (k+1)$.

Hence $n^5 - n$ is divisible by 5 for any non-negative integer n. $\square$.

21. Prove that for $r \in \mathbb{R}$, $r \neq 1$ and for all integers n, $\sum_{j=0}^{n} r^j = \frac{r^{n+1} - 1}{r - 1}$

We will proceed by induction. **Base Case:** $n = 0$. Then $\sum_{j=0}^{n} r^j = r^0 = 1$. While $\frac{r^{n+1} - 1}{r - 1} = \frac{r - 1}{r - 1} = 1$ (provided $r \neq 1$).

Induction Step: Suppose that $\sum_{j=0}^{k} r^j = \frac{r^{k+1} - 1}{r - 1}$ and consider $\sum_{j=0}^{k+1} r^j$.

Then $\sum_{j=0}^{k+1} r^j = \sum_{j=0}^{k} r^j + r^{k+1} = \frac{r^{k+1} - 1}{r - 1} + r^{k+1} = \frac{r^{k+1} - 1}{r - 1} + \frac{r^{k+1}(r - 1)}{r - 1}$
$= \frac{r^{k+1} - 1 + r^{k+2} - r^{k+1}}{r - 1} = \frac{r^{k+2} - 1}{r - 1}$ $\square$.

22. Prove that for all $n \geq 2$, $\sum_{k=1}^{n} \frac{1}{k^2} < 2 - \frac{1}{n}$

We will proceed by induction. **Base Case:** $n = 2$. Then $\sum_{k=1}^{2} \frac{1}{k^2} = \frac{1}{1} + \frac{1}{4} = \frac{5}{4} < 2 - \frac{1}{2} = \frac{3}{2}$.

Induction Step: Suppose $\sum_{k=1}^{n} \frac{1}{k^2} < 2 - \frac{1}{n}$ and consider $\sum_{k=1}^{n+1} \frac{1}{k^2}$.

$$\sum_{k=1}^{n+1} \frac{1}{k^2} = \sum_{k=1}^{n} \frac{1}{k^2} + \frac{1}{(n+1)^2} < 2 - \frac{1}{n} + \frac{1}{(n+1)^2} = 2 - \frac{n^2 + 2n + 1}{n(n+1)^2} + \frac{n}{n(n^2 + 2n + 1)}$$

$$= 2 - \frac{n^2 + n + 1}{n(n+1)^2} = 2 - \frac{n^2 + n}{n(n+1)^2} - \frac{1}{n(n+1)^2} = 2 - \frac{1}{n+1} - \frac{1}{n(n+1)^2} < 2 - \frac{1}{n+1}$$

Hence for all $n \geq 2$, $\sum_{k=1}^{n} \frac{1}{k^2} < 2 - \frac{1}{n}$ $\square$.

23. Prove that $n! < n^n$ whenever $n > 1$.

Base Case: $n = 2$. Then $2! = 2$ while $2^2 = 2$. Then $2 < 4$.

Induction Step: Suppose $k! < k^k$ and consider $(k+1)!$

Lemma: If $0 < x < y$, then $x^n < y^n$ for all n.
Base Case: When $n = 1$, $x < y$.
Inductive step: If $x^n < y^n$, then $x \cdot x^n = x^{n+1} < x \cdot y^n$. Similarly, $x \cdot y^n < y \cdot y^n = y^{n+1}$
Hence $x^{n+1} < y^{n+1}$. $\square$.
Applying the Lemma, Since $k < k + 1$, $k^k < (k+1)^k$. By the induction hypothesis, $(k+1)! = k!(k+1) < k^k(k+1)$.
Thus $(k+1)! < (k+1)^k(k+1) = (k+1)^{k+1}$ $\square$.

24. Prove that for all n, $\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$

Base Case: $n = 1$. Then $\sum_{k=1}^{1} \frac{1}{(2k-1)(2k+1)} = \frac{1}{(1)(3)} = \frac{1}{3}$, while $\frac{n}{2n+1} = \frac{1}{2+1} = \frac{1}{3}$

Induction Step: Suppose $\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$ and consider $\sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)}$.

Using the induction hypothesis, $\sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1} + \frac{1}{(2n+1)(2n+3)} = \frac{n(2n+3)}{(2n+1)(2n+3)} + \frac{1}{(2n+1)(2n+3)}$

$= \frac{2n^2+3n+1}{(2n+1)(2n+3)} = \frac{(2n+1)(n+1)}{(2n+1)(2n+3)} = \frac{n+1}{2n+3} = \frac{n+1}{2(n+1)+1}$ $\square$.