Math 210
Exam 3 Review Sheet

Section 10.1 - Graphs

• Understand the definition of a graph, and be clear on the difference between simple graphs, multigraphs, pseudographs, directed graphs, etc.
• Given a graph, be able to classify it according to the categories given above.
• Understand how various types of graphs can be used to model real world applications, and be able to choose the most appropriate type of model for a given application.

Section 10.2 - Graph Terminology and Special Types of Graphs

• Understand the definition of adjacency, edge incidence, vertex degree (in-degree and out-degree in directed graphs) and other basic graph terminology.
• Given a graph $\Gamma = (V, E)$, be able to find the cardinality of vertex and edge sets and the degrees of vertices (in-degree and out-degree in directed graphs).
• Memorize the Handshaking Theorem, and be able to use it to prove that an undirected graph has an even number of vertices of odd degree.
• Know the special simple graphs: K_n, C_n, W_n, Q_n, and $K_{m,n}$. Be able to draw examples of these classes of graphs and be able to compute the number of vertices and edges in these classes of graphs.
• Understand the definition of a bipartite graph and be able to prove whether or not a given graph is bipartite.
• Be able to use bipartite graphs to solve matching problems.
• Understand how to form subgraphs of a graph, the union of two graphs, and how to find the complement of a simple graph.

Section 10.3 - Representing Graphs and Graph Isomorphism

• Understand how to represent both directed and undirected graphs using Adjacency Lists, Adjacency Matrices, and Incidence Matrices.
• Understand the definition of graph isomorphism. Also, know the definition of a graph invariant, and key examples of graph invariants ($|E|, |V|$, degree sequences, etc.).
• Given a pair of graphs, be able to prove whether or not the given pair are isomorphic.

Section 10.4 - Connectivity

• Know the definition of a path of length n, a circuit, the sequence of a path, the vertex sequence of a path in a simple graph, and a directed path.
• Understand how paths can be interpreted in various graph models.
• Understand what it means for a graph to be connected, strongly connected, and weakly connected. Also, given a specific graph, be able to determine whether or not it is connected, and if not, be able to find its connected components (or strongly connected components).
• Understand the path length is a graph invariant, and be able to use this fact in graph isomorphism proofs.

Section 10.5 Euler and Hamilton Paths

• Understand the definition of Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Also understand how these are used in graph models.
• Given a specific (di)graph, be able to determine whether or not the graph has an Euler Path or an Euler Circuit by using key theorems. Also be able to find and label them in a graph when they exist.
• Given a specific (di)graph, be able to determine whether or not the graph has an Hamilton Path or a Hamilton Circuit by using key theorems. Also be able to find and label them in a graph when they exist.
• Understand the definition of and be able to identify cut vertices and bridge edges in a given graph.

Section 10.6 Shortest Path Problems

• Understand the definition of a weighted graph and the definition of a shortest path between vertices in a weighted graph.
• Understand Dijkstra's algorithm and be able to use it to find the shortest path between vertices in a weighted graph.
• Understand the Traveling Salesman problem and the issues which make it difficult to solve efficiently.
• Understand how to use a brute force algorithm in order to solve the TSP.
• Understand some of the practical applications of the shortest path problem and the TSP.

Section 10.8: Graph Coloring

• Know the definitions of a vertex coloring of a graph, and the (vertex) chromatic number of a graph.
• Be able to find the (vertex) chromatic number of a given graph.
• Know the four color theorem and be able to find the dual graph of a planar map *or* the dual graph of a planar graph.
• Be able to use graph vertex colorings to solve applications problems (like scheduling and radio frequency assignments).
• Know the definitions of an edge coloring of a graph, and the (edge) chromatic number of a graph.
• Be able to find the edge chromatic number of a given graph.