Math 210
Gateway Quiz 1 - Version 1 **Name:**______________________________
Solutions

1. (10 points) Write the **negation** of the following symbolic statement. Your answer should be written in fully simplified form (i.e. so that no negation is outside a quantifier and any negation symbols should only modify a single variable or predicate).

   $\exists x \forall y\, [(P(x,y) \wedge Q(x,y)) \rightarrow R(x,y)]$

   The simplified negation can be found as follows:

   $\neg \exists x \forall y\, [(P(x,y) \wedge Q(x,y)) \rightarrow R(x,y)]$

   $\equiv \forall x \exists y\, \neg\, [(P(x,y) \wedge Q(x,y)) \rightarrow R(x,y)]$.

   Recall the standard negation of a conditional is $\neg\,(p \rightarrow q) \equiv p \wedge \neg q$, so we have:

   $\equiv \forall x \exists y\, [(P(x,y) \wedge Q(x,y)) \wedge \neg R(x,y)]$.

2. (10 points) Translate the given statement into symbolic form. Be sure to define all predicates used. Use the variable a with domain $D = \{a : a \text{ is an animal }\}$ as your main variable. Then, negate the symbolic statement. Finally, rewrite the **negated** statement clearly in plain English.

   There is an animal that is not fluffy but which does make a good pet.

   We use the following predicates: $F(a):$ animal a is fluffy. $G(a):$ animal a makes a good pet.

   From this, we translate the original statement as follows: $\exists a\, [\neg F(a) \wedge G(a)]$.

   We then find the simplified negation: $\neg \exists a\, [\neg F(a) \wedge G(a)] \equiv \forall a \neg\, [\neg F(a) \wedge G(a)]$

   $\equiv \forall a\, [F(a) \vee \neg G(a)]$ (using De Morgan's Law and Double Negation)

   Finally, we translate the simplified negation back into standard English:

   Every animal is fluffy or does not make a good pet.