Math 210
Gateway Quiz 2 - Version 1 **Name:**______________________________

1. (20 points) Use mathematical induction to prove the following: $\sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{n(n+1)(n+2)}{6}$

   Base Case: $n = 1$. Then $\sum_{i=1}^{1} \frac{i(i+1)}{2} = \frac{1(2)}{2} = 1$, while $\frac{n(n+1)(n+2)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1$ so the Base Case is verified.

   Induction Step: Suppose $\sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{n(n+1)(n+2)}{6}$ and consider $\sum_{i=1}^{n+1} \frac{i(i+1)}{2}$. We must show that $\sum_{i=1}^{n+1} \frac{i(i+1)}{2} = \frac{(n+1)(n+2)(n+3)}{6}$

   Using the induction hypothesis, $\sum_{i=1}^{n+1} \frac{i(i+1)}{2} = \sum_{i=1}^{n} \frac{i(i+1)}{2} + \frac{(n+1)(n+2)}{2} = \frac{n(n+1)(n+2)}{6} + \frac{(n+1)(n+2)}{2}$

   $= \frac{n(n+1)(n+2)}{6} + \frac{3(n+1)(n+2)}{6} = \frac{n(n+1)(n+2) + 3(n+1)(n+2)}{6} = \frac{(n+1)(n+2)(n+3)}{6}$.

   Where the last step consists of factoring out the common factors $(n+1)$ and $(n+2)$ and regrouping. $\square$.