

Definitions are one of the fundamental building blocks of mathematical proofs. By necessity, definitions must be formed using other (usually more basic) words. Since this process cannot continue backward indefinitely, we must agree to understand the meaning and usage of some words contextually (or, more formally, we declare them to be undefined terms). Mathematics relies on precise, accurately worded definitions written in terms of previously defined terms or undefined terms. Creating good, useful, and understandable definitions is difficult but essential.

**Instructions:** This is a group activity. You must work together with your assigned group to answer these questions. Each of the following definitions have at least one significant flaw. Your job is to find the flaws and correct them to arrive at a precise definition for each term.

1. A number is **even** if it is a number that is a multiple of two.
2. A number is **odd** if it is a number that is not even.
3. A number is divisible by an integer  $k$  if it can be written as a product  $kx$ .
4. A **perfect square** is any number that is the square of some other number.
5. A number  $p$  is prime if it is not evenly divisible by any integers other than one and itself.
6. A **rational number** is any number that can be written in the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers.
7. An **irrational** number is any number that has the form  $\sqrt{n}$  for a number  $n$  that is not a perfect square.
8. A **set** is any collection of elements drawn from some universal set.
9. A function  $f : A \rightarrow B$  is **onto** if for every  $a \in A$  there is at least one  $b \in B$  so that  $f(a) = b$ .
10. A function  $f : A \rightarrow B$  is **one-to-one** if whenever  $f(a) = b_1$  and  $f(a) = b_2$ , it must be the case that  $b_1 = b_2$ .