

**Instructions:** This is a group activity. You must work together with your assigned group to answer these questions. Write a paragraph proof for each of the following propositions. The following definitions will be helpful to you as you work to write these proofs:

**Definitions:**

- An integer  $n$  is **even** if  $n = 2k$  for some integer  $k \in \mathbb{Z}$ .
- An integer  $n$  is **odd** if  $n = 2k + 1$  for some integer  $k \in \mathbb{Z}$ .
- Two integers have the **same parity** if they are both even or they are both odd. Otherwise they have **opposite parity**.
- Given two integers  $m$  and  $n$ , we say that  $m$  **divides**  $n$ , written  $m|n$  if  $n = km$  for some  $k \in \mathbb{Z}$ . In this case, we say that  $m$  is a **divisor** of  $n$  and that  $n$  is a **multiple** of  $m$ .

1. **Proposition 1:** If  $n$  is odd, then  $n^2$  is odd.

2. **Proposition 2:** If  $n$  is even, then  $n^2$  is even.

3. **Proposition 3:** If  $n$  is odd, then  $n^2 + 3n + 5$  is odd.

4. **Proposition 4:** If  $n$  is even, then  $mn$  is even.

5. **Proposition 5:** Let  $a$ ,  $b$ , and  $c$  be integers. If  $a|b$  and  $b|c$ , then  $a|c$ .

6. **Proposition 6:** Let  $a$ ,  $b$ , and  $c$  be integers. If  $a|b$  and  $a|c$ , then  $a|(b + c)$ .

7. **Proposition 7:** Let  $a$ ,  $b$ ,  $c$  and  $d$  be integers. If  $a|b$  and  $c|d$ , then  $ac|bd$ .

8. **Proposition 8:** If two integers have the same parity, then their sum is even.

9. **Proposition 9:** If  $n^2$  is even, then  $n$  is even.

10. **Proposition 10:** If  $a$  does not divide  $bc$ , then  $a$  does not divide  $b$ .