Math 311
Equivalence Relations Activity **Name:**________________________________

Recall:

- A relation R on a set A is called **reflexive** if $(a, a) \in R$ for every $a \in A$.
- A relation R on a set A is called **symmetric** if whenever $(a, b) \in R$, $(b, a) \in R$ as well.
- A relation R on a set A is called **antisymmetric** if whenever $(a, b) \in R$ and $(b, a) \in R$, then $a = b$.
- A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.

Definitions:

- A relation R on a set A is called an **equivalence relation** if it is reflexive, symmetric, and transitive.
- Two elements that are related by an equivalence relation R are called equivalent. If two elements a and b are equivalent, we denote this by $a \sim b$, or, if we want to explicitly refer to the relation under which they are equivalent, we write $a \sim_R b$.
- We use $[a]_R$ to denote the **equivalence class of** a **under the relation** R. That is, $[a]_R = \{b : b \in A \text{ and } a \sim_R b\}$.

Examples: For each of the following relations, determine whether or not the relation described is an equivalence relation. If it is, describe the equivalence classes of elements as clearly as you can. For those that are not equivalence relations, state (and justify) which properties fail to hold.

1. Let $A = \mathbb{R}$. Then we define $a\&b$ as follows. $a\&b$ if and only if $|a| = |b|$.

2. Let $A = \mathbb{R}$. Then we define $a \sim_1 b$ as follows. $a \sim_1 b$ if and only if $a = b$.

3. Let $A = \mathbb{R}$. Then we define $a \sim_2 b$ as follows. $a \sim_2 b$ if and only if $a \geq b$.

4. Let $A = \mathbb{R}$. Then we define $a \sim_3 b$ as follows. $a \sim_3 b$ if and only if $a \,|\, b$.

5. Let $A = \mathbb{Z}$. Then we define $a \equiv b(mod\, m)$ as follows. $a \equiv b(mod\, m)$ if and only if m divides $a - b$. begin by looking at the specific case $m = 3$ before considering the general case.