Math 311
Exam 1 - Practice Problems

1. For each of the following, translate the given argument into symbolic form. Then determine whether or not the given argument is valid by either writing a 2-column proof for the argument or by finding a counterexample.

   (a)
   If I work hard every day then I will get a promotion at work.
   If I do not get a promotion at work then I will not be able to afford my house payment.
   I can afford my house payment.

   Therefore I work hard every day.

   (b)
   If gas is expensive and parking is inconvenient then I will take the bus to school.
   If I take the bus to school then I will not be able to take a night class.
   I am taking a night class.

   Therefore gas is not too expensive or parking is convenient.

2. Determine whether or not the given argument is valid by translating it into symbolic form and identifying its form as a known valid argument form or a known fallacy.

   (a)
   If I want to go out on Saturday night then I need to study for my exam during the afternoon.
   I do not want to go out on Saturday night.

   Therefore I do not need to study for my exam during the afternoon.

   (b)
   If I study for my exam Saturday during the afternoon then I can go out on Saturday night.
   I do not go out on Saturday night.

   Therefore I did not study for my exam Saturday afternoon.

   (c)
   If I do not study for my exam then I will not get a good grade on it.
   I got a good grade on my exam.

   Therefore I studies for my exam.

3. Prove that each of the following arguments are valid by constructing a 2-column proof.

   (a)
   If you are unhappy with the results of the election then the candidate you voted for did not win.
   If the candidate you voted for did not win, then she will run again next election.
   The candidate you voted for is not running next election

   Therefore you are happy with the election results

   (b)
   Everyone who is brave and intelligent majors in mathematics.
   Tony is not a math major.
   Tony is brave.

   Therefore Tony is not intelligent.

4. Prove that the product of three odd integers is odd.

5. Prove that if n is an integer and that $n^2 + 11$ is even, then n is odd.

6. Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

7. Show that the following three statements are all equivalent:

   (i) x is rational

   (ii) $\frac{x}{2}$ is rational

   (iii) $x + 1$ is rational.

8. Prove or Disprove: The sum of two irrational numbers is irrational.
9. Prove that there is an integer m such that $m^3 > 10^{10}$. Is your proof constructive or non-constructive?
10. Prove that given any two rational numbers $p < q$, there is a rational number r with $p < r < q$.
11. Prove that given a non-negative integer n, there is a unique non-negative integer m such that $m^2 \leq n < (m+1)^2$
12. Prove or disprove: Every non-negative integer can be written as the sum of at most 3 perfect squares.
13. Which digits can occur as the final digits of the cube of an integer? Justify your answer.
14. Use a paragraph to show that $(B - A) \cup (C - A) = (B \cup C) - A$.
15. Use a paragraph to show that $A \cup (A \cap B) = A$.
16. Use a paragraph proof to show that $A - B = A \cap \overline{B}$.