## **Rules of Inference**

| Rule of Inference            | Tautology   | Name                       |
|------------------------------|---|----------------------------|
| p                            | $[p \land (p \to q)] \to q$                         | Modus Ponens               |
| $p \rightarrow q$            |   | (or Law of Detachment)     |
| $\therefore q$               |   |                            |
| $\neg q$                     | $[\neg q \land (p \to q)] \to \neg p$               | Modus Tollens              |
| $p \rightarrow q$            |   | (or Law of Contraposition) |
| $\therefore \neg p$          |   |                            |
| $p \rightarrow q$            | $[(p \to q) \land (q \to r)] \to (p \to r)$         | Hypothetical Syllogism     |
| $q \rightarrow r$            |   | (or Law of Syllogism)      |
| $\therefore p \rightarrow r$ |   |                            |
| $p \lor q$                   | $[(p \lor q) \land \neg p] \to q$                   | Disjunctive Syllogism      |
| $\neg p$                     |   |                            |
| $\therefore q$               |   |                            |
| p                            | $p \to (p \lor q)$                                  | Addition                   |
| $\therefore p \lor q$        |   |                            |
| $p \wedge q$                 | $(p \land q) \to p$                                 | Simplification             |
| $\therefore p$               |   |                            |
| p                            | $[(p) \land (q)] \to (p \land q)$                   | Conjunction                |
| q                            |   |                            |
| $\therefore p \land q$       |   |                            |
| $p \lor q$                   | $[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$ | Resolution                 |
| $  \neg p \lor r$            |   |                            |
| $\therefore q \lor r$        |   |                            |

## Rules of Inference for Quantified Statements

| Rule of Inference              | Name                       |
|--------------------------------|----------------------------|
| $\forall x P(x)$               | Universal Instantiation    |
| $\therefore P(c)$              |                            |
| P(c) for an arbitrary $c$      | Universal Generalization   |
| $\therefore \forall x P(x)$    |                            |
| $\exists x P(x)$               | Existential Instantiation  |
| $\therefore P(c)$ for some $c$ |                            |
| P(c) for some $c$              | Existential Generalization |
| $\therefore \exists x P(x)$    |                            |