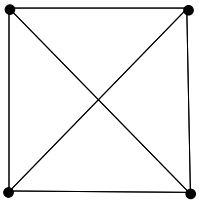
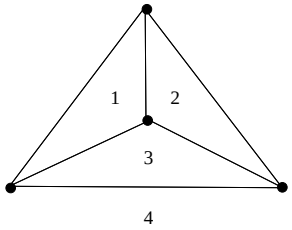
Math 311
Planar Graphs Activity **Name:**_____

Definition: A graph is called **planar** if it can be drawn in the plane without any edges crossing (where a crossing of edges is the intersection of the arcs representing them at a point other then a common endpoint). Such a drawing is called a **planar representation** of the graph.

Example: Consider K_4, the complete graph on four vertices.





Note that there are two drawings of K_4 in the figure above. This first is **not** a planar representation, while the second **is** a planar representation. To be clear, K_4 is planar since it has a planar representation. However, if we draw a less than optimal representation of a given graph, the drawing may not be planar even though the graph is planar.

You should also notice that numbers have been added to the drawing. These numbers correspond to the **regions** present in this planar representation of the graph. Notice that K_4 has 4 vertices, 6 edges, and 4 regions (the fourth region is the easiest to miss – it corresponds to the "outside" region in the drawing).

Exercise: For each graph, determine whether or not the graph is planar either by finding a planar representation for the graph or by showing that no planar representation exists. For those that are planar, find v, e, and r (the number of vertices, edges, and regions) for the graph. Do you see a relationship between these three quantities?

1. K_3

2. C_8

3. $K_{2,2}$

4. W_3

5. W_4

6. $K_{2,3}$

7. Q_3

8. W_5

9. $K_{3,3}$

10. K_5