Math 311
Portfolio Proofs - Version 2

The purpose of this document is to provide a list of proofs that are eligible to be added to your portfolio of proofs for the course. Each of you is expected to complete 50 points worth of portfolio proofs by the end of the course. When you submit a portfolio proof, I will grade it. If your proof is well written and essentially correct, I will award you points for the proof toward your portfolio total, and you will be able to add it to your portfolio. If the proof is incorrect or unclear, you will be expected to rewrite it and resubmit it for grading. Eventually, portfolio problems will expire (I will give a sunset date for each proof as the course progresses), but fear not, new proofs will be added so you will continue to have opportunities to earn portfolio points. Please note that some proofs will be grouped together in categories. When this happens, you will only be eligible to earn points for **one** proof in a given category.

1. Let $f(x) = mx + b$ be an arbitrary linear function.
   (a) Prove that $f(x)$ is one-to-one.
   (b) Prove that $f(x)$ is onto.
   (c) Find a formula for $f^{-1}(x)$.
2. Use a Direct Proof to show the following: Let a , b, and c be integers. If $a|b$ and $a|c$, then $a|(b+c)$.
3. Use Proof by Contraposition to prove the following: Let a , b, and c be integers. If a does not divide bc, then a does not divide b.
4. Use Proof by Contradiction to prove the following: Let m, n be integers. If $m+n$ is even, then m and n have the same parity.
5. Prove or Disprove: If the numbers 4, 5, 6, 7, 8, 9 are placed around a circle in some order, then there must be two consecutive numbers whose sum is at least 15.
6. Let $a, b \in \mathbb{N}$. Prove that there are unique integers q and r for which $a = bq + r$, with $0 \leq r < b$.
7. Prove the following using general element arguments: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
8. Suppose that R and S are relations on a set B. Further suppose that R and S are both transitive. Prove **two** of the following:
   (a) Is $R \cup S$ transitive? Justify your answer.
   (b) Is $R \cap S$ transitive? Justify your answer.
   (c) Is $R - S$ transitive? Justify your answer.
   (d) Is $R \circ S$ transitive? Justify your answer.
   (e) Is $\overline{R}$ transitive? Justify your answer.
9. Prove The following: (Corollary 3 to Euler's Formula) If Γ is a connected planar simple graph with e edges and v vertices, with $v \geq 3$ and no circuits of length three, then $e \leq 2v - 4$.
10. Prove, using only the properties given in the abstract definition of a Boolean Algebra B, that $x \vee x = x$ and $x \wedge x = x$ for every element x in B.
11. Determine whether or not it is possible to tile a 10 by 10 checkerboard using straight tetrominoes. **Note:** a straight tetromino is a 4 by 1 rectangle (see pp. 103-106 in your textbook). Be sure to fully justify your answer.
12. TBA