

Math 310

Project 3 Handout

Due: Tuesday, November 10th by 4:00pm

**Instructions:** This project is designed to give you an opportunity to explore some of the concepts from class in a little more depth. You may work with at most one other student on this assignment. If you decide to work with another student, you may turn in a combined paper with both your names listed.

1. In  $(1, 2, 3) - \text{Misère NIM}$ , the game begins with a pile of  $N$  stones. On their turn, a player can take either 1, 2, or 3 stones. However, unlike the version presented in class, in this case, the player that takes the last stone **loses** the game.
  - (a) (1 point) Which player has a winning strategy if the game starts with 7 stones?
  - (b) (3 points) Do a complete analysis of  $(1, 2, 3) - \text{Misère NIM}$  as the game is defined above. That is, determine which player has a winning strategy for any value of  $N$  [Hint: split into cases].
  - (c) (4 points) Generalize your results to  $(1, 2, 3, \dots, k) - \text{Misère NIM}$  in which a player can take either 1, 2, 3, ..., or  $k$  stones.
  - (d) (4 points) Do a complete analysis of  $(2, 3) - \text{Misère NIM}$  in which a player can take either 2 or 3 stones.