

Math 311

Project 4 Handout

Due: Tuesday, November 17th by 4:00pm

**Instructions:** This project is designed to give you an opportunity to explore some of the concepts from class in a little more depth. You may work with at most one other student on this assignment. If you decide to work with another student, you may turn in a combined paper with both your names listed.

1. Recall that the notation  $[a, b]$  is used to denote the closed interval from  $a$  to  $b$ . That is,  $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$ . Similarly,  $(a, b)$  is used to denote the open interval from  $a$  to  $b$ . That is,  $(a, b) = \{x \in \mathbb{R} : a < x < b\}$ .
  - (a) (5 points) Use a formal element argument to prove that  $[a, b] = \bigcap_{n=1}^{\infty} \left(a - \frac{1}{n}, b + \frac{1}{n}\right)$ . This demonstrates that every closed interval can be written as the intersection of infinitely many open intervals.
  - (b) (3 points) Find an expression that represents the open interval  $(a, b)$  as the union of infinitely many closed intervals.
  - (c) (5 points) Use a formal element argument to prove that the expression you found in part (b) is equal to  $(a, b)$ . This demonstrates that every open interval can be written as the union of infinitely many closed intervals.