Math 311
Project 7 Handout
Due: Wednesday, December 9th by 4:00pm

Instructions: This project is designed to give you an opportunity to explore some of the concepts from class in a little more depth. You may work with at most one other student on this assignment. If you decide to work with another student, you may turn in a combined paper with both your names listed.

Definitions:

• A **standard checkerboard** is an 8×8 grid of squares with the squares colored black and white in an alternating pattern along each row of squares.
• A **domino** is a 1×2 grid of squares.
• A **tiling** of a checkerboard is an arrangement of dominoes that covers every square of the board with no pieces overlapping and no portion of a piece overhanging the edge of the board.

1. (2 points) Prove that it is possible to tile a standard checkerboard using dominoes.
2. (2 points) Prove that it is not possible to tile a 9×9 checkerboard using dominoes.
3. (3 points) A straight tri-omino is a 1×3 grid of squares. Prove that it is not possible to tile a standard 8×8 checkerboard using straight tri-ominoes, but that it is possible to tile a 9×9 checkerboard using tri-ominoes.
4. (5 points) A straight k-omino is a $1 \times k$ grid of squares. For which k is it possible to tile a standard 8×8 checkerboard using straight k-ominoes? Justify your answer.
5. (8 points) A "left-ell" k-omino is a straight $(k-1)$-omino with a single additional square attached immediately to the right of the bottom-most square in the the $(k-1)$-omino (when the k-omino is placed vertically). Say as much as you can about which $n \times n$ square checkerboards can be tiled using a single given type of "left-ell" k-ominoes. Say as much as you can about which $m \times n$ rectangular checkerboards can be tiled using a single given type of "left-ell" k-ominoes.

   Note: Since not many of you made significant progress on part 3 of project 6, I am going to given you another opportunity yo try this part:

   Project 6 Part 3 – (4 points) Find an order under which $\mathbb{Z} \times \mathbb{Z}$ is well ordered. Justify your answer.