Math 311
Relations Activity **Name:**______________________________

Let A and B be sets. Recall that a **binary relation** (or, more briefly, a **relation**) from A to B is a subset of $A \times B$. Elements of a relation R are usually written as ordered pairs of the form (a, b). Here, $R \subseteq A \times B$, and $(a, b) \in R$. We sometimes write $a\,R\,b$, meaning that a is related to b via R. If $(a, b) \notin R$, then we say a is not related to b. In the special case that $A = B$, then a relation $R \subseteq A \times A$ is called a relation on the set A.

Properties of Relations:

1. A relation R on a set A is called **reflexive** if $(a, a) \in R$ for every $a \in A$.

2. A relation R on a set A is called **symmetric** if whenever $(a, b) \in R$, $(b, a) \in R$ as well.

3. A relation R on a set A is called **antisymmetric** if whenever $(a, b) \in R$ and $(b, a) \in R$, then $a = b$.

4. A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.

Examples: Let $A = \{0, 1, 2, 3\}$, and consider the following relations on A:

- $R_1 = \{(0, 1), (1, 2), (2, 3), (3, 3)\}$
- $R_2 = \{(0, 0), (0, 1), (1, 1), (1, 2), (2, 2), (2, 3), (3, 3)\}$
- $R_3 = \{(0, 0), (1, 1), (2, 2), (3, 3)\}$
- $R_4 = \{(1, 2), (2, 1), (1, 1)\}$
- $R_5 = \{(0, 0), (0, 1), (1, 0), (1, 1), (2, 2), (1, 3), (3, 1), (3, 3)\}$

1. Which of these relations are reflexive?

2. Which of these relations are symmetric?

3. Which of these relations are antisymmetric?

4. Which of these relations are transitive?

5. Suppose A is the vertex set for a directed graph and the relation R is the set of directed edges. Draw the graphs of R_1, R_2, R_3, R_4, and R_5.

6. Examine the graphs that you draw in problem #5. Explain how you could tell directly from the graph that a relation R is reflexive. How could you tell from the graph that R is symmetric? Antisymmetric? Transitive?

7. Find each of the following:

   (a) $R_1 \cup R_3$

   (b) $R_1 \cap R_3$

   (c) $R_1 - R_3$

   (d) $R_3 - R_1$

   (e) $\overline{R_3}$.

8. Find the adjacency matrix M_{R_i} for R_1, R_2, R_3, R_4, and R_5.