Math 311
Uniqueness Proofs Activity

Instructions: This is a group activity. You must work together with your assigned group to answer these questions. Write a proof for each of the following propositions. Begin each proof by outlining the cases that you will consider.

Completing a uniqueness proof for a statement of the form: $\exists!\, x\, P(x)$ requires two steps. First, we must show **existence** – that is, we use either a constructive or a non-constructive existence proof to proof the related statement $\exists\, x\, P(x)$. Second, we must show **uniqueness** – that is, we must show that there is **only one** instance of x that works. To accomplish the second part, we will usually start by supposing that there are two instances x_1 and x_2, and we demonstrate that $x_1 = x_2$, verifying the uniqueness of the instance.

1. **Proposition 1:** Let $f(n) = n + 2$. Show that for each $k \in \mathbb{Z}$ there is a unique $\ell \in \mathbb{Z}$ such that $f(\ell) = k$

2. **Proposition 2:** Show that for any irrational number r, there is a unique integer n such that $|r - n| < \frac{1}{2}$. .