Math 450
Exam 3 Review Sheet

Section 3.4: Cubic Spline Interpolation

• Understand that **piecewise linear** approximation and **cubic splines** are examples of a technique called piecewise-polynomial approximation.
• Know the conditions that a cubic spline must satisfy, and be able to find the system of equations that the cubic spline must satisfy for a specific example (for both "free" and "clamped" endpoints).
• Given information about values and some coefficients for a cubic spline, be able to solve to find the other coefficients.
• Know and be able to apply the error formula for a clamped cubic spline to find an upper bound on the error when using a clamped spline to approximate the value of a given function.

Section 4.1: Numerical Differentiation

• Know and be able to use **forward difference** and **backward difference** formulas to approximate the derivative of a function at a given point. Also be able to find an upper bound on the error in these approximations.
• Be able to derive the **forward difference** and **backward difference** approximation formulas and their error terms by differentiating the first Lagrange Polynomial and its error term.
• Know and be able to apply **three-point** and **five-point** formulas to approximate the derivative of a function at a given point. Also be able to find an upper bound on the error in these approximations.
• Given a table of values for a given function, be able to select and apply the most accurate method for approximating the derivative for each of the given x values. Also be able to use first order derivative approximations in order to recursively estimate values for higher order derivatives.

Section 4.2: Richardson's Extrapolation

• Understand the derivation of Richardson's Extrapolation procedure: namely, that given an approximation method whose error term is a power series in h, one can recursively define new approximation formulas with higher degree order convergence than the previous approximation formula.
• Given an approximation formula and its error series, be able to apply Richardson's Extrapolation in order to find a new approximation formula with higher degree order of convergence than the original approximation formula.
• Be able to apply Richardson's Extrapolation to find accurate numerical approximations of the derivative of a function based on provided data. Also be able to find the related error term and be able to use the error term to find an upper bound on the error of your approximation.

Section 4.3: Elements of Numerical Integration

• Understand and be able to apply the Midpoint Rule, the Trapezoid Rule, and Simpson's Rule to approximate the value of the definite integral of a function over a given interval.
• Know the error terms for the Trapezoid Rule and for Simpson's Rule and be able to apply them to find an upper bound on the error of your numerical approximations of definite integrals.
• Understand the definition of the **degree of accuracy** or **precision** of a numerical integration technique. Also, be able to apply this definition (using appropriately chosen functions) to solve for the coefficients for a numerical integration technique.
• Be able to carry out the integration necessary to derive the error term for the Trapezoid Rule.

Section 4.4: Composite Numerical Integration

• Understand how subdividing the interval for a given definite integral can be used to develop composite numerical integration methods based on each of the single interval numerical integration methods developed in section 4.3.
• Know the basic formulas and error terms for Composite Simpson's Rule and the Composite Trapezoid Rule.
• Be able to apply Composite Simpson's Rule and the Composite Trapezoid Rule to both approximate the value of a given definite integral and to find an upper bound on the error of your approximation.
• Understand the how the formulas for Composite Simpson's Rule and the Composite Trapezoid Rule are derived, including their error terms.
• Be able to use the error formulas for Composite Simpson's Rule and the Composite Trapezoid Rule in order to determine the step size and number of intervals necessary in order to approximate a given definite integral to within a desired error bound.

Section 4.5: Romberg Integration

• Understand Romerg Integration and how it is derived by applying Richardson's Extrapolation to an existing numerical integration technique.
• Be able to apply Romberg integration to find accurate numerical approximations of a given definite integral.
• Understand the effect of Richardson's Extrapolation on the error term of a numerical integration technique and be able to find the error term for a Romberg Integration Method.