

Math 450
Programming Assignment 2
Due: Tuesday October 25th

1. Let $f(x) = \ln x$.
 - (a) Find (either directly or using Maple or a program of your own) the first five non-zero Taylor Polynomial Approximations of $f(x) = \ln x$ centered at $c = 1$.
 - (b) Use the polynomials $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, and $P_5(x)$ you found above to approximate $\ln 1.5$. Find the absolute error of each approximation.
 - (c) Use Maple (or some other graphing utility) to produce a plot containing $f(x)$ along with $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, and $P_5(x)$ [All of these functions should be labeled and displayed on the **same** plot].
2.
 - (a) Write a program that, when given as input a set of data values $(x_0, y_0), (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, outputs the Lagrange Polynomial of degree n passing through these points. [To make things easier, since we know the underlying function being used, you may set your program up so that the input is in the form: $(n, f(x), x_0, x_1, \dots, x_n)$. Since the data below is equally spaced, you could choose to write a program accepting equally spaced data taking input: $(n, f(x), x_0, h = \Delta x)$].
 - (b) Let $f(x) = \ln x$ and let $x_0 = 1, x_1 = 1.2, x_2 = 1.4, x_3 = 1.6, x_4 = 1.8, x_5 = 2.0$. Use your program to find the Lagrange Polynomials $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$.
 - (c) Use the polynomials $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$ you found above to approximate $\ln 1.5$. Find the absolute error of each approximation.
 - (d) Use Maple to produce a plot containing $f(x)$ along with $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$ [All of these functions should be displayed on the **same** plot]
3.
 - (a) Write a program that, when given input $(n, f(x), f'(x), x_0, x_1, \dots, x_n)$, computes the associated Hermite Polynomial. [You may use either method that we know for computing Hermite Polynomials as the basis for your program. Notice that I am allowing you to give $f'(x)$ as part of the input].
 - (b) Let $f(x) = \ln x$ and let $x_0 = 1, x_1 = 1.4, x_2 = 1.8$. Use your program to find the Hermite polynomial that agrees with both $f(x)$ and $f'(x)$ at x_0, x_1 , and x_2 .
 - (c) Use the polynomial you found above to approximate $\ln 1.5$. Find the absolute error of the approximation.
 - (d) Use Maple (or some other graphing utility) to produce a plot containing $f(x)$ and the polynomial you found above.
4.
 - (a) Given $x_0 = 1, x_1 = \frac{4}{3}, x_2 = \frac{5}{3}, x_3 = 2$ find the free spline agreeing with $f(x)$ at these values.
 - (b) Given $x_0 = 1, x_1 = \frac{4}{3}, x_2 = \frac{5}{3}, x_3 = 2$ find the clamped spline agreeing with $f(x)$ at these values. [Note: unlike the previous parts, you may find these via direct computation in Maple rather than using a program written in Maple or another programming language.]
 - (c) Use the piecewise defined functions you found above to approximate $\ln 1.5$. Find the absolute error of each approximation.
 - (d) Use Maple (or some other graphing utility) to produce a plot containing $f(x)$ and the functions you found above.

Extra credit: Write a program to find the free spline fitting data input as: $(n, f(x), x_0, x_1, \dots, x_n)$.
5.
 - (a) Which of the approximating functions you found above approximate $f(x) = \ln x$ most accurately at $x = 1.5$? Justify your answer.
 - (b) Which of the approximating functions you found above approximate $f(x) = \ln x$ most accurately throughout the entire interval $[1, 2]$? Justify your answer.