

Math 450
Programming Assignment 4
Due: Friday, December 9th

1. Write programs that implements each of the following numerical methods. Your algorithm should take as input a function $f(t, y)$, an interval $[a, b]$, an initial condition $y(a) = \alpha$, and a step size h . For the higher order Taylor Methods, you may perform the additional differentiations externally.
 - (a) Euler's Method
 - (b) Taylor's Method of order 2.
 - (c) Taylor's Method of order 4.
 - (d) The Midpoint Method.
 - (e) Modified Euler's Method.
 - (f) Heun's Method.
 - (g) Runge-Kutta Order 4.

2. Given the initial value problem $y' = -ty + 4ty^{-1}$, $0 \leq t \leq 1$, $y(0) = 1$, use each of your algorithms to approximate the value of $y(1)$ using each of the algorithms you coded above:
 - (a) For $h = 0.1$.
 - (b) For $h = 0.01$

3.
 - (a) Verify that the solution to this differential equation is: $y(t) = \sqrt{4 - 3e^{-t^2}}$
 - (b) Find the absolute error of each of your estimates using $h = 0.1$.
 - (c) Find the absolute error of each of your estimates using $h = 0.01$.
 - (d) Compute the ratio of each pair absolute errors you found. What, if anything, do these ratios indicate about the rate of convergence of each numerical algorithm?