Math 476 - Abstract Algebra 1
Day 11 Group Assignment **Name:**______________________________

Cyclic Groups and the Center of a Group

Definition 22.7 Let G be a group. The **center** of G is the set $Z(G) = \{a \in G : ab = ba \text{ for all } b \in G\}$. In other words, the set of all elements that commute with every element in G.

1. Find the center of each of the following groups:

   (a) $\mathbb{Z}_{10}$

   (b) The group G of symmetries of an equilateral triangle

   (c) The group G of symmetries of a square

2. Let G be a group with identity element e.

   (a) Is e in $Z(G)$? Explain.

   (b) Is $Z(G)$ closed under the operation in G? Prove your answer.

   (c) If $a \in Z(G)$, is $a^{-1} \in Z(G)$? Prove your answer.

   (d) If $Z(G)$ a subgroup of G? Explain. Is $Z(G)$ Abelian?

Based on your work above, complete the statement of the following theorem:

Theorem 22.10 Let G be a group. The center of G is a(n) ____________ ____________.

Definition 22.13 Let G be a group, and let $a \in G$. The **cyclic subgroup generated by** a, denotes $\langle a \rangle$, is defined by $\langle a \rangle = \{a^n : n \in \mathbb{Z}\}$, or, if the group is written using additive notation, $\langle a \rangle = \{na : n \in \mathbb{Z}\}$.

Definition 22.14 A group G is **cyclic group** id $G = \langle a \rangle$ for some $a \in G$.

3. Explain why a cyclic group, as defined in Definition 22.13 satisfies each of the properties of a group.

4. In each of the following you are given a group G and an element a. List the elements of $\langle a \rangle$.

   (a) $G = \mathbb{Z}_{10}$, $a = [3]$

   (b) $G = \mathbb{U}_{10}$, $a = [3]$

5. Determine whether or not the following groups are cyclic groups.

   (a) $\mathbb{Z}_n$ (under the addition operation)

   (b) $\mathbb{Z}$ (under the addition operation)

   (c) $\mathbb{R}$ (under the addition operation)

   (d) The group G of symmetries of an equilateral triangle

   (e) U_{10} (under the multiplication operation)