 Name:________________________________

Group Isomorphism Theorems

Theorem 30.13 (The First Isomorphism Theorem) Let G and H be groups and let φ from G to H be a group **homomorphism**. Then $G/Ker(\varphi) \cong Im(\varphi)$.

Note: We will not go through the proof of this theorem in complete detail. The main technique at the heart of the proof is constructing an isomorphism between the two groups (in general, defining a specific function mapping one group to another and then showing that this function is an isomorphism is how we go about showing that two groups are isomorphic). In this setting, the function that we use is:

Let $\Phi : G/K \to Im(\varphi)$ be given by $\Phi(aK) = \varphi(a)$.

1. Let $G = \mathbb{Z}_{24}$, $H = \mathbb{Z}_8$ and let $\varphi : G \to H$ be defined by $\varphi([m]_{24}) = [6m]_8$.

   (a) Find $K = Ker\ \varphi$ and $R = Im(\varphi)$.

   (b) Make a multiplication table for the the elements of the group G/K.

   (c) Make a multiplication table for the the elements of the group $R = Im(\varphi)$.

   (d) Let $\Phi : G/K \to Im(\varphi)$ be given by $\Phi(aK) = \varphi(a)$. Prove that Φ is a group isomorphism.

2. Let $G = D_6$ and $K = \langle r, R^3 \rangle = \{I, R^3, r, rR^3\}$, and $N = \langle r, R^2 \rangle = \{I, R^2, R^4, r, rR^2, rR^4\}$.

(a) Find the elements of $K \cap N$.

(b) Let $KN = \{kn : k \in K, n \in N\}$. Find the elements of KN. Is $KN < D_6$?

(c) Find the elements of $K/(K \cap N)$

(d) Find the elements of KN/N

(e) What relationship do you observe between $K/(K \cap N)$ and KN/N?

Theorem 30.16 (The Second Isomorphism Theorem) Let G be a group, K a subgroup of G, and $N \triangleleft G$. then $K/(K \cap N) \cong KN/N$.