Math 476 - Abstract Algebra 1
Day 27 Group Assignment **Name:**______________________________

Theorem 7.10 Let R be a ring, and suppose that both 0 and $0'$ are zero elements for R. Then $0 = 0'$.

Theorem 7.11 Let R be a ring, and suppose that both 1 and $1'$ are identities for R. Then $1 = 1'$.

1. The goal for this activity is to outline a proofs for Theorem 7.10 and Theorem 7.11
   (a) Let a be an element of R. What must $a + 0$ and $a + 0'$ be equal to, and why?

   (b) Use your answer to part (a) to equate $a + 0$ and $a + 0'$.

   (c) What axiom or theorem, along with your answer to part (b), allows you to conclude that $0 = 0'$?

   (d) Explain why the strategy you used for Theorem 7.10 would not provide a valid proof for Theorem 7.11

   (e) Prove Theorem 7.11 by evaluating $1 \cdot 1'$ in two different ways.

Definition 7.13: Let R be a ring with identity, and let $x \in R$. An element $y \in R$ is said to be a **multiplicative inverse** of x provided that $xy = 1 = yx$.

Definition 7.14: Let R be a ring with identity. An element $x \in R$ is said to be a **unit** provided that R contains a multiplicative inverse for x. In other words, $x \in R$ is a unit if and only if there exists $y \in R$ such that $xy = 1 = yx$.

Theorem 7.15: Let R be a ring, and let $x \in R$. Suppose that both y and y' are additive inverses for x. Then $y = y'$.

Proof: Let R be a ring, let $x \in R$, and let y and y' be inverses for x. Then $x+y = 0$ and $x+y' = 0$. Then $x+y = x+y'$. Therefore, by commutativity of addition, $y + x = y' + x$. Hence, by Theorem 7.3, $y = y'$. $\square$.

Theorem 7.16: Let R be a ring with identity, and let $x \in R$ be a unit. Suppose that both y and y' are multiplicative inverses for x. Then $y = y'$.

2. Give a complete proof for Theorem 7.16.

Theorem 7.17: Let R be a ring with identity, and let z be a unit in R. For all $x, y \in R$, if $xz = yz$, then $x = y$. Similarly, if $zx = zy$, then $x = y$.

3. Provide a specific example that shows that this result may not hold if z is not a unit.

Definition 7.18 Let R be a ring. An element x is said to be a **zero divisor** if $x \neq 0$, and $xy = 0$ or $yx = 0$ for some nonzero $y \in R$.

Theorem 7.19 Let R be a ring. The following statements are equivalent:

- R contains no zero divisors.
- For all $x, y \in R$, if $xy = 0$, then $x = 0$ or $y = 0$.
- For all $x, y \in R$, if $xy = 0$ and $x \neq 0$, then $y = 0$.

Theorem 7.20 Let R be a ring, and let z be a nonzero element of R that is not a zero divisor. For all $x, y \in R$, if $xz = yx$, then $x = y$.

Proof: See p. 85 in your textbook

Theorem 7.21 Let R be a ring with identity, and let $x \in R$ be a unit. Then x is not a zero divisor. That is, if $xy = 0$ or $yx = 0$ for some $y \in R$, then $y = 0$.

Proof: See p. 85 in your textbook

Definition 7.22: An **integral domain** is a commutative ring with identity that contains no zero divisors.

Definition 7.23: A **field** is a commutative ring with identity in which every nonzero element has a multiplicative inverse.

4. Given an example of a field and an example of an integral domain that is not a field.

5. In the space below, write out Theorem 7.26 in your textbook.