Math 476 - Abstract Algebra 1
Day 2 Group Assignment **Name:**____________________________

Instructions: Complete as many of the following problems as you can during class time.

1. Discuss the pros and cons of each of the following potential definitions of the additive inverse of an integer x.

   (a) The additive inverse of x is $-x$.

   (b) The additive inverse of x is $0 - x$.

   (c) The additive inverse of x is an integer y such that $x + y = 0$.

   (d) The additive inverse of x is $(-1)x$.

2. Let a, b, and c be integers.

   (a) Prove that $-(ac) = (-a)c$

   (b) Prove that $-(a + b) = -a - b$.

3. In the space provided, write out the Ordering Axioms of the Integers (from page 8 in your textbook):

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4. One of our axioms for the integers states that multiplication distributes over addition.

   (a) Write out a precise statement that shows what it would mean for addition to distribute over multiplication.

   (b) Does addition distribute over multiplication? Provide a proof or a counterexample to justify your answer.

5. Section I-1 of your text defines the additive inverse of an integer and uses this to define integer subtraction.

   (a) State a formal definition for the multiplicative inverse for in integer a in $\mathbb{Z}$.

   (b) A **unit** is an integer that has a multiplicative inverse. Determine all **units** in $\mathbb{Z}$.

The Division Algorithm: Let a and b be integers with $a > 0$. Then there exist unique integers q and r such that $b = aq + r$ and $0 \leq r < a$.

We call q the **quotient** and r the **remainder**.

6. Which of the following sets has a least element?

   - $A = \{1, 2, 3, 4\}$
   - $B = \{x \in \mathbb{Z} : x < 4\}$
   - $C = \{x \in \mathbb{Z} : x > 4\}$
   - $D = \{x \in \mathbb{W} : x > 4\}$
   - $E = \{x \in \mathbb{W} : x < 4\}$

7. Let a and b be integers with $a > 0$. Let $S = \{x \in \mathbb{Z} : x \geq 0 \text{ and } x = b - am \text{ for some } m \in \mathbb{Z}\}$.

   (a) Let $a = 5$ and $b = 43$. Find at least 5 different elements of S. Which integer appears to be the least element of S?

   (b) Repeat part (a) with $a = 10$ and $b = -58$.

   (c) Prove that if $b \geq 0$, then $b \in S$ (regardless of the value of a).

   (d) Suppose $b < 0$. For what values of m will $b - am$ be an element of S?

The Well Ordering Principle: Every nonempty subset of the whole numbers contains a least element.