Math 476 - Abstract Algebra 1
Day 31 Group Assignment **Name:**______________________________

Isomorphism and Invariants

Definition 10.5 Let R and S be rings. An **isomorphism** is a bijective function $\varphi : R \to S$ such that for all $x, y \in R$, $\varphi(x+y) = \varphi(x)+\varphi(y)$ and $\varphi(xy) = \varphi(x)\varphi(y)$. If there exists an isomorphism from R to S, then R is said to be **isomorphic to** S.

1. Suppose that R_1 and R_2 are given by the following tables:

$(R_1)+$	γ	α	δ	β
γ	α	γ	β	δ
α	γ	α	δ	β
δ	β	δ	γ	α
β	δ	β	α	γ

$(R_1)\cdot$	γ	α	δ	β
γ	α	α	γ	γ
α	α	α	α	α
δ	γ	α	β	δ
β	γ	α	δ	β

$(R_2)+$	i	j	k	ℓ
i	i	j	k	ℓ
j	j	k	ℓ	i
k	k	ℓ	i	j
ℓ	ℓ	i	j	k

$(R_2)\cdot$	i	j	k	ℓ
i	i	i	i	i
j	i	j	k	ℓ
k	i	k	i	k
ℓ	i	ℓ	k	j

(a) Let the function $\varphi : R_1 \to R_2$ be defined via $\varphi(\gamma) = k$, $\varphi(\alpha) = i$, $\varphi(\delta) = \ell$, and $\varphi(\beta) = j$. Show that φ is an isomorphism.

(b) Let the function $\phi : R_1 \to R_2$ be defined via $\phi(\gamma) = i$, $\phi(\alpha) = j$, $\phi(\delta) = k$, and $\phi(\beta) = \ell$. Show that φ is **not** an isomorphism.

2. Let $M = \left\{ \begin{bmatrix} x & 0 \\ -x & 0 \end{bmatrix} : x \in \mathbb{R} \right\}$.

(a) Show that M, as defined above, is a subring of $\mathcal{M}_{2\times 2}(\mathbb{R})$.

(b) Think of a familiar ring that M seems to be isomorphic to. You do not need to formally prove that this ring is isomorphic to M, but you should give a reasonable explanation of why you believe that they are isomorphic.

(c) What would you need to show in order to formally prove that these two rings are isomorphic? You only need to state the steps you would use – you do not need to actually show them.

Definition 10.10 Let S and T be sets. A mapping $f : S \to T$ is **well-defined** is $f(a) = f(b)$ whenever $a = b$ in S (does this definition look familiar?).

3. Let f be a map from $\mathbb{Q}$ to $\mathbb{Z}$ defined by $f\left(\frac{a}{b}\right) = ab$. Determine whether or not this map is well defined.