Math 476 - Abstract Algebra 1
Group Assignment

Name: ____________________

Presentation Problems

For the problems on this assignment, you can work on proving them in groups and on your own between now and class on Monday. Once you feel you have a good argument, you can volunteer to present a problem to the rest of the class.

1. Let G be a group. Prove that if $(ab)^2 = a^2b^2$, then $ab = ba$.
2. Consider the permutation $\sigma = (13256)(23)(46512)$.
    (a) Write σ in disjoint cycle notation.
    (b) Find the order of σ.
    (c) Determine whether σ is odd or even.
3. Let $\varphi : G \to H$ be a group isomorphism. Prove that G is cyclic if and only if H is cyclic.
4. Find a subgroup of $\mathbb{Z}_{12} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_{15}$ that has order 9.
5. Let $G = \{ax^2 + bx + c : a, b, c \in \mathbb{Z}_3\}$ where the operation is polynomial addition. Is $G \cong \mathbb{Z}_3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3$? Justify your answer.
6. Let $G = \mathbb{Z}_4 \oplus \mathbb{Z}_{12}$, where the operation is addition.
    (a) Find $H = \langle (2, 2) \rangle$
    (b) Show that H is a normal subgroup of G.
    (c) Build the Cayley Table for G/H.
7. Let $\varphi : G \to H$ be a group homomorphism. Prove that if $K \lhd G$, then $\varphi(K)$ is normal in H.
8. Let $R = \mathbb{Z}_3 \oplus \mathbb{Z}_6$. Find all units in R.
9. Suppose that a is an idempotent in a commutative ring R with identity 1_R. Show that $1_R - a$ is also an idempotent.
10. Let I and J be ideals of a ring R. Must $I \cap J$ be an ideal? Provide either a proof or a specific counterexample.
11. Show that $4x^2 + 6x + 3$ is a unit in $\mathbb{Z}_8$.
12. Let $\varphi : \mathbb{Z}_4 \to \mathbb{Z}_{12}$ be given by $\varphi([k]_4) = [3k]_{12}$. Determine whether or not φ is a ring homomorphism.