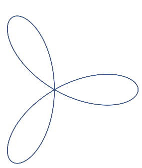
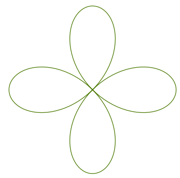
Math 476 - Abstract Algebra 1
Day 7 Group Assignment **Name:**________________________________

Symmetry





1. Identify and describe the different types of symmetry that are present in each of the figures above.

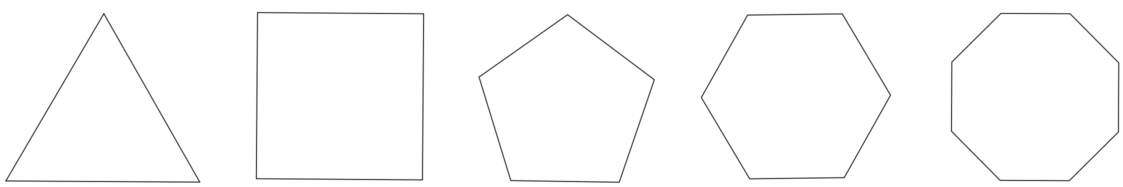
Definition 19.2 A **rigid motion** in a plane is a bijective function $f : \mathcal{P} \to \mathcal{P}$ such that, for all $p, q \in \mathcal{P}$, $d(p,q) = d(f(p), f(q))$, where d is the distance function for the given plane.

Recall: When $\mathcal{P} = \mathbb{R}^2$ and d is the standard Euclidean distance, rigid motions can be broken down in to the following categorization: translations, rotations, reflections, and glide reflections.

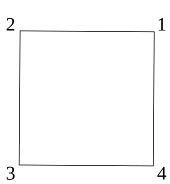
Definition 19.3 A **symmetry** of a geometric object O is a rigid motion f such that $f(O) = O$.

Claim: The composition of any two symmetries is a symmetry.

Our goal for the remainder of this activity is to understand the symmetries of the following regular polygons:



One nice way to describe the symmetries of a regular polygon is to label the vertices of the polygon and then use **2-row permutation notation**. Consider the symmetries of a square as an example. They can be represented as follows.



$$I = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \qquad D_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$$

$$D_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix} \qquad V = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

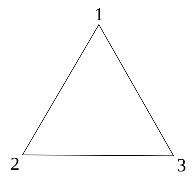
$$H = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \qquad R_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$R_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \qquad R_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

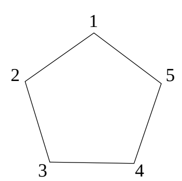
$\circ$	I	V	H	D_1	D_2	R_1	R_2	R_3
I	I	V	H	D_1	D_2	R_1	R_2	R_3
V	V	I	R_2	R_1	R_3	D_1	H	D_2
H	H	R_2	I	R_3	R_1	D_2	V	D_1
D_1	D_1	R_3	R_1	I	R_2	H	D_2	V
D_2	D_2	R_1	R_2	R_2	I	V	D_1	H
R_1	R_1	D_2	D_1	V	H	R_2	R_3	I
R_2	R_2	H	V	D_2	D_1	R_3	I	R_1
R_3	R_3	D_1	D_2	H	V	I	R_1	R_2

2. Spend some time understanding and verifying the operation table for the symmetries of the square given above.

3. Find permutation representations and Build a Composition Operation Table for the symmetries of an equilateral triangle [Building a physical model might be helpful].



4. Find permutation representations and Build a Composition Operation Table for the symmetries of a regular pentagon.



5. If you have time, do the same for a regular hexagon and/or octagon.