Math 450
Examples of Computing Interpolating Polynomials

restart

Consider the following data values:

$x0 := -1 : x1 := 0 : x2 := 1 : x3 := 2 :$

$y0 := -1 : y1 := 5 : y2 := 7 : y3 := 11 :$

Lagrange's Method:

$$P3 := x \to \frac{y0 \cdot (x-0) \cdot (x-1) \cdot (x-2)}{(-1-0) \cdot (-1-1) \cdot (-1-2)} + \frac{y1 \cdot (x+1) \cdot (x-1) \cdot (x-2)}{(0+1) \cdot (0-1) \cdot (0-2)} + \frac{y2 \cdot (x+1) \cdot (x-0) \cdot (x-2)}{(1+1) \cdot (1-0) \cdot (1-2)} + \frac{y3 \cdot (x+1) \cdot (x-0) \cdot (x-1)}{(2+1) \cdot (2-0) \cdot (2-1)}$$

$$x \to -\frac{1}{6}\, y0\, x\, (x-1)\, (x-2) + \frac{1}{2}\, y1\, (x+1)\, (x-1)\, (x-2) - \frac{1}{2}\, y2\, (x+1)\, x\, (x-2) + \frac{1}{6}\, y3\, (x+1)\, x\, (x-1) \quad \textbf{(1)}$$

$P3(x)$

$$\frac{1}{6}\, x\, (x-1)\, (x-2) + \frac{5}{2}\, (x+1)\, (x-1)\, (x-2) - \frac{7}{2}\, (x+1)\, x\, (x-2) + \frac{11}{6}\, (x+1)\, x\, (x-1) \quad \textbf{(2)}$$

$\xrightarrow{\text{simplify symbolic}}$

$$x^3 - 2\, x^2 + 3\, x + 5 \quad \textbf{(3)}$$

Neville's Method:

$Q00 := y0;\ Q10 := y1;\ Q20 := y2;\ Q30 := y3$

$$-1$$
$$5$$
$$7$$
$$11 \quad \textbf{(4)}$$

$$Q11 := x \to \frac{(x - x0) \cdot Q10 - (x - x1) \cdot Q00}{x1 - x0};\ Q21 := x \to \frac{(x - x1) \cdot Q20 - (x - x2) \cdot Q10}{x2 - x1};\ Q31 := x \to \frac{(x - x2) \cdot Q30 - (x - x3) \cdot Q20}{x3 - x2}$$

$$x \to \frac{(x - x0)\, Q10 - (x - x1)\, Q00}{x1 - x0}$$
$$x \to \frac{(x - x1)\, Q20 - (x - x2)\, Q10}{x2 - x1}$$
$$x \to \frac{(x - x2)\, Q30 - (x - x3)\, Q20}{x3 - x2} \quad \textbf{(5)}$$

$Q11(x);\ Q21(x);\ Q31(x)$

$$6\, x + 5$$
$$2\, x + 5$$
$$4\, x + 3 \quad \textbf{(6)}$$

$$Q22 := x \to \frac{(x - x0) \cdot Q21(x) - (x - x2) \cdot Q11(x)}{x2 - x0};\ Q32 := x \to \frac{(x - x1) \cdot Q31(x) - (x - x3) \cdot Q21(x)}{x3 - x1}$$

$$x \to \frac{(x - x0)\ Q21(x) - (x - x2)\ Q11(x)}{x2 - x0}$$

$$x \to \frac{(x - x1)\ Q31(x) - (x - x3)\ Q21(x)}{x3 - x1} \quad \textbf{(7)}$$

$Q22(x);\ Q32(x)$

$$\frac{1}{2}\ (x + 1)\ (2\ x + 5) - \frac{1}{2}\ (x - 1)\ (6\ x + 5)$$

$$\frac{1}{2}\ x\ (4\ x + 3) - \frac{1}{2}\ (x - 2)\ (2\ x + 5) \quad \textbf{(8)}$$

$\xrightarrow{\text{simplify symbolic}}$

$$-2\ x^2 + 4\ x + 5 \quad \textbf{(9)}$$

$\xrightarrow{\text{simplify symbolic}}$

$$x^2 + x + 5 \quad \textbf{(10)}$$

$$Q33 := x \to \frac{(x - x0) \cdot Q32(x) - (x - x3) \cdot Q22(x)}{x3 - x0}$$

$$x \to \frac{(x - x0)\ Q32(x) - (x - x3)\ Q22(x)}{x3 - x0} \quad \textbf{(11)}$$

$Q33(x)$

$$\frac{1}{3}\ (x + 1)\ \left(\frac{1}{2}\ x\ (4\ x + 3) - \frac{1}{2}\ (x - 2)\ (2\ x + 5)\right) - \frac{1}{3}\ (x - 2)\ \left(\frac{1}{2}\ (x + 1)\ (2\ x + 5) - \frac{1}{2}\ (x - 1)\ (6\ x + 5)\right) \quad \textbf{(12)}$$

$\xrightarrow{\text{simplify symbolic}}$

$$x^3 - 2\ x^2 + 3\ x + 5$$

Newton's Divided Difference Method:

i	x_i	$f[x_i]$	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$	$f[x_{i-3}, x_{i-2}, x_{i-1}, x_i]$
0	-1	$f0 := -1$ -1 **(13)**	$f01 := \frac{f1 - f0}{x1 - x0}$:	$f012 := \frac{f12 - f01}{x2 - x0}$:	$f0123 := \frac{f123 - f012}{x3 - x0}$:
1	0	$f1 := 5$ 5 **(14)**	$f12 := \frac{f2 - f1}{x2 - x1}$:	$f123 := \frac{f23 - f12}{x3 - x1}$:	
2	1	$f2 := 7$ 7 **(15)**	$f23 := \frac{f3 - f2}{x3 - x2}$:		
3	2	$f3 := 11$ 11 **(16)**			

$$R := x \to f0 + f01 \cdot (x - x0) + f012 \cdot (x - x0) \cdot (x - x1) + f0123 \cdot (x - x0) \cdot (x - x1) \cdot (x - x2)$$

$$x \to f0 + f01\ (x - x0) + f012\ (x - x0)\ (x - x1) + f0123\ (x - x0)\ (x - x1)\ (x - x2) \tag{17}$$

$R(x)$

$$5 + 6\,x - 2\,(x + 1)\,x + (x + 1)\,x\,(x - 1) \tag{18}$$

$\xrightarrow{\text{simplify symbolic}}$

$$x^3 - 2\,x^2 + 3\,x + 5 \tag{19}$$