

Math 311
Exam 1 Review Sheet

Propositional Logic and Quantifiers

- Understand the definition of a proposition and be able to determine whether or not a given statement is a proposition.
- Understand the definition and truth tables of the logical operators \neg , \wedge , \vee , \Rightarrow , and \Leftrightarrow
- Be able to build the truth table of any compound proposition.
- Be able to translate back and forth between English statements and symbolic logical propositions.
- Understand the various ways of expressing a conditional statement in English (e.g. necessary, sufficient, only if, whenever, ...). Also know the difference between a conditional statement and its converse, inverse, and contrapositive statement.
- Understand what it means for two propositions to be logically equivalent and be able to prove the logical equivalence of a pair of propositions by building truth tables.
- Understand the what it means for a proposition to be a *tautology*, a *contradiction*, or a *contingency*.
- Be able to prove that a proposition is a tautology.
- Be able to negate propositions using De Morgan's Laws and other equivalences.
- Understand the definition of a predicate involving one or more variables.
- Understand the definition of the universal quantifier, the existential quantifier, and the uniqueness quantifier.
- Be able to determine the truth value of statements involving predicates and/or quantifiers.
- Understand the definition of a *counterexample* and how to use them to show that a statement is false.
- Understand how to find a "useful denial" of a statement involving predicates and quantifiers.
- Understand "hidden quantifiers" and how "mixed quantifiers" can work together to form a statement and the importance of the order of quantification in a statement.
- Be able to determine the truth value of statements involving multiple and "mixed" quantifiers.
- Be able to negate statements involving multiple and "mixed" quantifiers.

Basic Proof Techniques

- Understand and be able to apply logical argument forms such as *modus ponens*.
- Be able to read a mathematical statement and understand the "Given" and the "Goal" of the statement.
- Be able to write proofs by utilizing: Direct proof, Proof by Contraposition, and Proof by Contradiction.
- Memorize and be able to apply the definitions of odd numbers, even numbers, rational numbers, prime numbers, absolute value, and divisibility.
- Be able to use the Basic Properties of Integers and Real Numbers to prove related properties (you are **not** expected to memorize these).
- Understand and be able to show or make use of "closure properties" of integers, rational numbers, and real numbers.
- Understand and be able to carry out a "for all" proof.
- Understand and be able to carry out an "existence" proof.
- Understand and be able to carry out a "uniqueness" proof.
- Understand and be able to carry out proofs of "biconditional" statements.
- Understand and be able to carry out proofs of "or" statements.
- Understand and be able to utilize Proof by Cases (the Method of Exhaustion).
- Understand how to rule out unnecessary cases (without loss of generality statements).
- Understand the role of counterexamples in disproving statements.
- Understand how to use "backwards reasoning" to help find a proof for a given statement.
- Understand how to form conjectures and how to use examples to investigate a statement.