Math 311
Final Exam Review Sheet

Part 1: New Content Since the Previous Exam

Congruences

• Understand the definition of congruence modulo n and be able to determine whether or not two given integers are congruent modulo n for some given n.
• Be able to prove that congruence modulo n is reflexive, symmetric, and transitive (and thus is an equivalence relation).
• Know the statement of Theorem 6.4.5.
• Be able to prove basic facts about congruence modulo n such as those outlined in Theorem 6.4.6.
• Understand the definition of a congruence class modulo n and be able to find the congruence of any integer for some given n.
• Know the statement of Theorem 6.5.5 and understand that congruence classes modulo n form a partition of $\mathbb{Z}$.
• Understand "canonical representatives" for congruence classes modulo m and how they are used to define $\mathbb{Z}_m$.
• Understand the definition of addition, subtraction, and multiplication in $\mathbb{Z}_m$. Be able to perform computations and build operation tables for a given value of m.
• Know the statement of Theorem 6.5.6 and how it related to the well-definedness of the operations in $\mathbb{Z}_m$.

Relations

• Know the definition of a (binary) relation and how it is represented as a set of ordered pairs.
• Be able to find the domain of a relation.
• Understand that every function is a relation but that not every relation is a function.
• Understand what it means for a relation to be reflexive, symmetric, antisymmetric, or transitive.
• Be able to determine whether a given relation on a set is reflexive, symmetric, antisymmetric, or transitive.
• Be able to combine relations to form new ones using union, intersection, complementation, or difference.
• Given a specific relation be able to represent is using a directed graph.
• Given a directed graph, be able to list the ordered pairs in the relation represented by the directed graph.
• Be able to recognize whether a relation on a set is reflexive, symmetric, antisymmetric, or transitive by looking at its directed graph.

Equivalence Relations

• Know the definition of an equivalence relation and be able to determine (and prove) whether a given relation is an equivalence relation.
• Understand the definition of the equivalence class of an element with respect to an equivalence relations on a set A and be able to find the equivalence class of a given element.
• Understand the relationship between equivalence classes and partitions of the underlying set. Be also to tell whether or not a given family of subsets is a partition, and if so, be able find the equivalence relation described by the partition.
• Given a directed graph, be able to determine whether the associated relation is an equivalence relation.

Partial Orderings

• Know the definition of a partial order and be able to prove whether a given relation is a partial order.
• Understand the definition of comparable and incomparable elements, and be able to determine whether a given pair of elements are comparable or incomparable with respect to a particular partial order.
• Understand definition of a total ordered set and a well ordered set.
• Be able to draw the Hasse diagram for a poset. Also be able to read off the ordered pairs in a partial order from its Hasse diagram.
• Understand the definition of maximal elements, minimal elements, greatest elements, least elements, upper bounds, lower bounds, least upper bounds, greatest lower bounds, and the definition of a lattice. Also be able to find these (when they exist) for elements in a given poset.

Part 2: Cumulative Content From Previous Exams
Mathematical Logic

• Understand the definition of a proposition and be able to determine whether or not a given statement is a proposition.
• Understand the definition and truth tables of the logical operators $\neg$, $\wedge$, $\vee$, $\Rightarrow$, and $\Leftrightarrow$
• Be able to build the truth table of any compound proposition.
• Be able to translate back and forth between English statements and symbolic logical propositions.
• Understand the various ways of expressing a conditional statement in English (e.g. necessary, sufficient, only if, whenever, ...). Also know the difference between a conditional statement and its converse, inverse, and contrapositive statement.
• Understand what it means for two propositions to be logically equivalent and be able to prove the logical equivalence of a pair of propositions by building truth tables.
• Understand the what it means for a proposition to be a *tautology*, a *contradiction*, or a *contingency*.
• Be able to prove that a proposition is a tautology.
• Be able to negate propositions using De Morgan's Laws and other equivalences.
• Understand the definition of a predicate involving one or more variables.
• Understand the definition of the universal quantifier, the existential quantifier, and the uniqueness quantifier.
• Be able to determine the truth value of statements involving predicates and/or quantifiers.
• Understand the definition of a *counterexample* and how to use them to show that a statement is false.
• Understand how to find a "useful denial" of a statement involving predicates and quantifiers.
• Understand "hidden quantifiers" and how "mixed quantifiers" can work together to form a statement and the importance of the order of quantification in a statement.
• Be able to determine the truth value of statements involving multiple and "mixed" quantifiers.
• Be able to negate statements involving multiple and "mixed" quantifiers.

Basic Proof Techniques

• Understand and be able to apply logical argument forms such as *modus ponens*.
• Be able to read a mathematical statement and understand the "Given" and the "Goal" of the statement.
• Be able to write proofs by utilizing: Direct proof, Proof by Contraposition, and Proof by Contradiction.
• Memorize and be able to apply the definitions of odd numbers, even numbers, rational numbers, prime numbers, absolute value, and divisibility.
• Be able to use the Basic Properties of Integers and Real Numbers to prove related properties (you are **not** expected to memorize these).
• Understand and be able to make use of "closure properties" of integers, rational and real numbers.
• Understand and be able to carry out a "for all" proof.
• Understand and be able to carry out an "existence" proof.
• Understand and be able to carry out a "uniqueness" proof.
• Understand and be able to carry out proofs of "biconditional" statements.
• Understand and be able to carry out proofs of "or" statements.
• Understand and be able to utilize Proof by Cases (the Method of Exhaustion).
• Understand how to rule out unnecessary cases (without loss of generality statements).
• Understand the role of counterexamples in disproving statements.
• Understand how to use "backwards reasoning" to help find a proof for a given statement.
• Understand how to form conjectures and how to use examples to investigate a statement.

Mathematical Induction

• Know the definition of a sequence along with standard sequence notation.
• Understand both explicitly and recursively defined sequences and be able to use them to write out the first few terms in a sequence.
• Understand summation notation and be able to compute summations.
• Understand the principle of mathematical induction and why it is a valid method of proof.
• Be able to prove the base case of a set of statements of the form $P(n)$ for all $n > 0$
• Be able to prove theorems using mathematical induction and strong induction.
• Be able to recognize errors in false induction "proofs".

Sets

• Understand the definitions of sets, elements, subsets, as well as symbolic notation for these terms.
• Understand both roster and set builder notation, and be able to determine whether a given set is well defined.
• Understand the definition of set equality and the definition of the cardinality of a set.
• Understand the definitions of the universal set, the empty set, and the power set of a set.
• Understand how to form the Cartesian Product of two sets.
• Understand the definition of the set operations: union, intersection, complementation, and set difference.
• Be able to determine which elements are in a set resulting from multiple set operations. Also be able to draw a Venn diagram representing such a set.
• Be able to prove statements and properties of sets using containment arguments.
• Understand how to take the union or the intersection of many sets at the same time (including infinite unions and intersections).

Functions

• Understand the definition of a function, including the definition of the domain, co-domain, and range of a function and be able to apply these definitions to specific functions. Also know how to find the implicit domain of a function.
• Understand and be able to apply the definitions of image and preimage (for both individual elements and sets of elements).
• Understand function equality and be able to determine whether or not a pair of functions are equal.
• Understand the definition of the graph of a function, and be able to draw the graph of a given function.
• Know the definition of a polynomial function, including the degree of a polynomial.
• Know and be able to apply the definition of the identity function on a set.
• Understand the definition of function composition and be able to find composite functions and to prove facts about composite functions.
• Know the definition of one-to-one and onto, and be able to determine whether or not a given function satisfies these definitions.
• Understand the definition of an inverse function, know the properties that a function must satisfy in order to have an inverse, and be able to find the inverse of a function or to determine whether or not a given pair of functions are inverses.