Math 311 - Introduction to Proof and Abstract Mathematics
Group Assignment # 18 **Name:**______________________________
Presentation Day!

General Instructions: On a semi-regular basis, instead of having a regular group work day, we will have a *Presentation Day*. On these days, you will still be given time to work on problems in groups. However, rather than writing up your solutions and turning them in for graded credit, you will be given the opportunity to present solutions to the assigned problems to your classmates on the board. You should still work cooperatively with your group, but a single individual from your group will be responsible for presenting the solution to the assigned problem. Whoever is ready first can present a problem, with one significant proviso – if there is a person in the class who has presented fewer total times to that point who is ready (or very nearly ready) to present a given problem, the student who has given fewer total presentations up to that point will be given the first opportunity to present.

Presentation Problems:

1. Give a **proof** or a **specific counterexample** for the following: If ℓ divides m and m divides n, then ℓ divides n.

2. Use the Principle of Mathematical Induction to prove that for all $n \geq 1$, $\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$

3. Use the Principle of Mathematical Induction to prove that for all $n \geq 5$, $n^2 < 2^n$.

4. Use mathematical induction to prove that $x^2 - 1$ is divisible by 8 whenever x is a *positive odd integer*.

5. Prove or disprove: Let a, b, c and d be integers. If $a|b$ and $c|d$, then $ac|bd$.

   [Claimed by Tim]

6. Prove that $x^2 + y^2 = 11$ has no integer solutions.

7. Prove or disprove: If a does not divide bc, then a does not divide b.

   [Claimed by Axum]

8. Formulate a conjecture about the decimal digits that appear as the final digit of the fourth power of an integer. Prove your conjecture using proof by cases.

9. Suppose that a fast food restaurant sells chicken nuggets in packs of 4, 7, or 9. What is the largest number of chicken nuggets that you **cannot** buy exactly (Fully justify your answer).

10. Suppose that a different restaurant sells chicken nuggets in packs of 4 or 15. What is the largest number of chicken nuggets that you **cannot** buy exactly (Fully justify your answer).

11. Prove Proposition 4.2.12

12. Prove $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

13. Prove $A \times (B \cap C) = (A \times B) \cap (A \times C)$

14. Prove or Disprove: If $A - C \subseteq B - C$ then $A \subseteq B$.

    [Claimed by Queenie]

15. Prove or Disprove: $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

    [Claimed by Tejendra]

16. Prove or Disprove: $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

    [Claimed by Jash]

17. Express the set $\{1\}$ as the intersection of a collection of distinct, non-empty intervals in $\mathbb{R}$ indexed by $\mathbb{Z}^+$.

18. Use the properties of Real Numbers to prove that given $a, b \in \mathbb{R}$, $ab = 0$ if and only if $a = 0$ or $b = 0$.

19. Let $\mathcal{P}$ be the set of all polynomial functions. Let $D : \mathcal{P} \to \mathcal{P}$ be the function given by $D(p(x)) = p'(x)$. That, the function that maps each function to its derivative. Each of the following is a separate presentation problem:

    (a) Find $im\, D$.

    (b) Let $\mathcal{L} = \{p(x) \in \mathcal{P} \,|\, p(x) = ax + b$. Find the preimage of $\mathcal{L}$ under D.

    (c) If D 1-1? Justify your answer.

20. Let $h(x) = \sqrt{x^2 + 1} - 5$. Find three different ways to express $h(x)$ as the composition of two functions (that is, find three distinct pairs of functions f and g so that $h = g \circ f$.

21. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f((x, y)) = x + y$. Show that f is not 1-1, but that f is onto.

22. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f((x, y)) = x + y$. Find a set $A_0 \subset \mathbb{R}^2$ so that $f : A_0 \to \mathbb{R}$ is a bijection.

23. Prove Corollary 5.4.10

24. This problem involves a famous set-theoretic concept called Russell's Paradox. According to Russell's definitions, a set A is called **normal** if A is *not* an element of itself. Similarly, a set is **abnormal** if it *is* an element of itself.

    (a) Give an example of a set that is normal.

    (b) Give an example of a set that is abnormal.

    (c) Let $\mathcal{N} = \{A \,|\, A \text{ is a set that is normal } \}$ and let $\mathcal{A} = \{A \,|\, A \text{ is a set that is abnormal } \}$.
    Is $\mathcal{N}$ a normal set or an abnormal set? How about $\mathcal{A}$? Explain how this leads to a paradox and comment on what caused things to go wrong.