Math 311 - Introduction to Proof and Abstract Mathematics
Group Assignment # 1 **Name:**______________________________
Due: Thursday, August 30th

1. Consider the following scenario: Once upon a time, a farmer went to a market and purchased a goat, and wolf, and a bag of cabbage (I am not sure why he wanted a wolf). On his way home, the farmer came to the bank of a river. Since the bridge had been destroyed in a flood the previous spring, he had to cross it by boat. Fortunately, he had left his trusty rowboat on the near bank. Unfortunately, the boat was quite small, so in crossing the river by boat, the farmer could carry only himself and a single one of his items: the goat, the wolf, or the bag of cabbage.

   - If left unattended together, the wolf would eat the goat
   - Similarly, if left unattended, and the goat would eat the cabbage.

   Help the farmer find a way to transport himself and all of his purchases to the far bank of the river in a way that will keep each item intact. You should find a specific method that the farmer can use, or demonstrate that no effective strategy exists. If a strategy does exist, try to find the most efficient method possible.

Definition 1.1.1 A **proposition** (or **statement**) is a sentence (i.e., it has both a subject and a verb) which has exactly one truth value; i.e. it is either true or false, but not both.

2. Determine whether or not each of the following are statements.

   (a) $2 + 2 = 4$

   (b) $2 + 2 = 5$

   (c) $2 + 2$

   (d) What is my name?

   (e) $x + 1 = 5$

   (f) This statement is false.

Basic logical connectives: Suppose P and Q represent statements. We define the basic logical connectives **conjunction**, **disjunction** and **negation** as follows:

- The *conjunction* of P and Q is the statement "P and Q", denoted $P \wedge Q$.
- The *disjunction* of P and Q is the statement "P or Q", denoted $P \vee Q$.
- The *negation* of P is the statement "not P", denoted $\neg P$.

Note: The statement P has two possible truth values (True or False – usually denoted by T or F), as does the statement Q. Therefore, the statement $P \wedge Q$ has four possible truth values: (T, T); (T, F); (F, T); and (F, F). Combining this with our sense of what "and" should mean, we have the following **truth table** for the statement $P \wedge Q$:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

3. In your own words, explain what this table tells us about the impact of the truth values of P and Q on the truth value of the compound expression $P \wedge Q$.

4. In the tables provided below, create truth tables for $P \vee Q$ and $\neg P$.

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

P	$\neg P$
T	
F	

5. Determine whether or not the following statements are true or false. Briefly justify your answers.

   (a) $2 + 2 = 5$ and $2^3 = 8$ (b) $2 + 2 = 5$ or $2^3 = 8$ (c) $2 + 2 = 5$ or $\neg(2^3 = 8)$

Note: Now that we know a bit about logical connectives and truth tables, we can begin building truth tables for more complicated expressions. For example, here is the truth table for the expression $\neg(P \wedge Q)$ – notice the role that "order of operations" plays in creating this table, and how the last column is formed by "negating" the previous column.

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

6. Create truth tables for the expression $\neg(P \vee Q)$ and the expression $\neg P \vee \neg Q$ (how many columns will be needed?).

Definition 1.1.6 Two statements are **logically equivalent** if they have the "same" truth table.

7. Give an example of two distinct statements that are logically equivalent.