Math 311 - Introduction to Proof and Abstract Mathematics
Group Assignment # 4 **Name:**______________________________
Presentation Day!

General Instructions: On a semi-regular basis, instead of having a regular group work day, we will have a *Presentation Day*. On these days, you will still be given time to work on problems in groups. However, rather than writing up your solutions and turning them in for graded credit, you will be given the opportunity to present solutions to the assigned problems to your classmates on the board. You should still work cooperatively with your group, but a single individual from your group will be responsible for presenting the solution to the assigned problem. Whoever is ready first can present a problem, with one significant proviso – if there is a person in the class who has presented fewer total times to that point who is ready (or very nearly ready) to present a given problem, the student who has given fewer total presentations up to that point will be given the first opportunity to present.

Definitions: Let $n \in \mathbb{Z}$.

(1) n is **even** if there exists an integer k such that $n = 2k$; i.e., n is even if and only if $(\exists k \in \mathbb{Z})[n = 2k]$.

(2) n is **odd** if there exists an integer k such that $n = 2k + 1$; i.e., n is odd if and only if $(\exists k \in \mathbb{Z})[n = 2k + 1]$.

(3) n is **prime** if $n > 1$ and the only positive integer factors of n are 1 and n; i.e., n is prime if $n > 1$ and $(\forall a, b \in \mathbb{N})[n = ab \Rightarrow (a = 1 \text{ or } b = 1)]$.

(4) An integer a **divides** an integer b, written $a|b$, if and only if there exists $n \in \mathbb{Z}$ such that $b = an$. In this case, we may also say that b *is divisible by* a.

Specific Instructions: Give a **proof** or a **specific counterexample** for each of the following.

1. The sum of two consecutive integers is odd.

2. The product of two consecutive integers is even.

3. The product of any two odd integers is odd.

4. If p and q are distinct prime numbers, then $pq + 1$ is prime.

5. There exist prime numbers p and q such that $pq + 1$ is prime.

6. For any positive integer n, $2^{2^n} + 1$ is prime.

7. If 6 divides n, then 3 divides n.

8. If k divides m and k divides n, then k divides $m + n$.

9. If ℓ divides m and m divides n, then ℓ divides n.

10. If k divides n^2, then k divides n.