Math 311
Portfolio Proofs - Version 5

The purpose of this document is to provide a list of proofs that are eligible to be added to your portfolio of proofs for the course. Each of you is expected to complete **ten (10) portfolio proofs** by the end of the course. When you submit a portfolio proof, I will grade it. If your proof is well written and essentially correct, I will award you five (5) points for the proof toward your portfolio total, and you will be able to add it to your portfolio. If the proof is incorrect or unclear, you will be expected to rewrite it and resubmit it for grading.

1. Use a Direct Proof to show the following: Let a , b, and c be integers. If $a|b$ and $a|c$, then $a|(b+c)$.

2. Use Proof by Contraposition to prove the following: Let a , b, and c be integers. If a does not divide bc, then a does not divide b.

3. Use Proof by Contradiction to prove the following: Let m, n be integers. If $m+n$ is even, then m and n have the same parity (that is, either they are both even or that are both odd).

4. Prove or Disprove: If the numbers 4, 5, 6, 7, 8, 9 are placed around a circle in some order, then there must be two consecutive numbers whose sum is at least 15.

5. Let $a, b \in \mathbb{N}$. Prove that there are unique integers q and r for which $a = bq + r$, with $0 \leq r < b$.

6. Use mathematical induction to prove the following: $\displaystyle\sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{n(n+1)(n+2)}{6}$

7. Let A, B, and C be sets. Show that $(A \cup C) - B = (A - B) \cup (C - B)$ using a paragraph proof that demonstrates that each side is a subset of the other side.

8. Let $f(x) = mx + b$ be an arbitrary linear function, with $m \neq 0$.
   (a) Prove that $f(x)$ is one-to-one.
   (b) Prove that $f(x)$ is onto.
   (c) Find a formula for $f^{-1}(x)$.

9. Let X, Y be sets and suppose that $f : X \to Y$ is a bijection. Prove that its inverse function exists and is unique.

10. Let $a_1, a_2, b_1, b_2 \in \mathbb{Z}$ and assume that $[a_1]_m = [a_2]_m$ and $[b_1]_m = [b_2]_m$. Prove that $[a_1 b_1]_m = [a_2 b_2]_m$.

11. Suppose that R and S are relations on a set X. Further suppose that R and S are both transitive. Complete **two** of the following:
    (a) Is $R \cup S$ transitive? Justify your answer.
    (b) Is $R \cap S$ transitive? Justify your answer.
    (c) Is $R - S$ transitive? Justify your answer.
    (d) Is $\overline{R}$ transitive? Justify your answer.

12. Consider the following relation on the set of polynomial functions $p : \mathbb{R} \to \mathbb{R}$. Two functions $f \sim g$ if and only if $f'(x) = g'(x)$. Prove that $\sim$ is an equivalence relation and decribe the equivalence class of the function $f(x) = x^2$.

13. Prove that $(\mathbb{N}, \leq)$ is a well ordered set.

14. Consider the poset $(A, |)$, where $A = \{1, 2, 4, 5, 6, 8, 12, 18, 20, 40\}$. Prove that this poset is **not** a lattice. Then make this poset into a lattice by adding additional elements to the set A (use the minimum number of additional elements).