Math 311
Presentation Problems

The following problems have been compiled to both aid you in preparing for the final exam and to serve as a list of presentation eligible problems during our last regular class meeting on Tuesday, December 11th. You are expected to give a clear and correct proof, and to be able to talk your classmates through your argument as you are presenting it on the board.

1. Find the logical negation of the statement: Everyone who passed the exam studied for the exam.

2. Determine whether or not the given argument is valid by either giving a proof or by finding a counterexample.

   If I work hard every day then I will get a promotion at work.
   If I do not get a promotion at work then I will not be able to afford my house payment.
   I can afford my house payment.
   ____
   Therefore I work hard every day.

3. Determine whether or not the given argument is valid by either giving a proof or by finding a counterexample.

   If I want to go out on Saturday night then I need to study for my exam during the afternoon.
   I do not want to go out on Saturday night.
   ____
   Therefore I do not need to study for my exam during the afternoon.

4. Determine whether or not the given argument is valid by either giving a proof or by finding a counterexample.

   If gas is expensive and parking is inconvenient then I will take the bus to school.
   If I take the bus to school then I will not be able to take a night class.
   I am taking a night class.
   ____
   Therefore gas is not too expensive or parking is convenient.

5. Prove that if n is an integer and that $n^2 + 11$ is even, then n is odd.

6. **Prove or Disprove:** Let a, b, and c be integers. If ab divides c, then a divides c.

7. Prove that for integers $n > 1$ that $n!$ is even. [Recall that $n! = n \cdot (n-1) \cdot \cdots \cdot 3 \cdot 2 \cdot 1$]

8. **Prove or Disprove:** The product of any 3 consecutive integers is a multiple of 3.

9. Determine whether or not there exist integers m and n such that $2m + 4n = 7$.

10. Prove that if r^2 is irrational, then r is also irrational.

11. Prove that $A - B = A \cap \overline{B}$.

12. Use Proof by contradiction to prove the following: If A and B are sets, then $A \cap (B - A) = \emptyset$.

13. **Prove or Disprove:** If A or B are sets satisfying $A \cup B = A \cap B$, then $A \cap \overline{B} = \emptyset$.

14. Prove that $n^5 - n$ is divisible by 5 for any non-negative integer n.

15. Prove that for all n, $\displaystyle\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$

16. Suppose that $f(x) = e^x$ and $g(x) = xe^x$. Use induction and the product rule to show that $g^{(n)}(x) = (x+n)e^x$ for all $n \geq 1$.

17. Let f be the function defined by $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ $f(m,n) = m^2 - n$. Determine whether or not f is a one-to-one. Also determine whether or not f is onto. Justify your answers.

18. Suppose $R = \{(x,y) \in \mathbb{R}^2 : y - x \text{ is an integer }\}$. Prove that R is an equivalence relation.

19. Define a relation R on $\mathbb{R}^2$ by $\{((x_1,y_1),(x_2,y_2)) \,|\, (x_1^2 + y_1^2) = (x_2^2 + y_2^2)\}$

    (a) Show that R is an equivalence relation.

    (b) Describe the equivalence classes of R.

20. Let A be the set of all people who attend MSUM. Define a relation on A as follows: $(a,b) \in \mathcal{R}$ if person a has completed at least as many credits as person b. Determine whether or not $\mathcal{R}$ is a poset.