Math 311
Project 2 Handout

Instructions: This handout is designed to describe a project on one of the "Additional Topics" that may to choose to complete in order to satisfy the Project component of your grade in the course. As noted in the Course Policies Handout, you can earn 25 points (or more) based on the progress you make on these projects. To help make sure that you have time to work on these, you will be given time in class to work on these rather than our standard Daily Group Work assignments during the next three class periods (starting on Thursday, November 29th). You are encouraged to work in groups on these projects. You should turn in **three** projects for grading. One should be completed by Tuesday, December 4th. Your second is due on Thursday, December 6th, and your third project must be completed by Tuesday, December 11th. You will have the opportunity to earn up to 10-12 points on each of these projects. Any points beyond 25 points will count as extra credit.

Project 2: Knight and Knave Logic Problems

"Knight" and "Knave" logic puzzles were invented (or perhaps popularized) by mathematician and philosopher Raymond Smullyan. These puzzles are all set on a fictional island whose inhabitants are either knights (who always tell the truth) or knaves (who always lie). "Solving" a puzzle means correctly identifying the identity of each of the inhabitants involved using logic and a set of statements made by the inhabitants. Note that it is possible to invent problems that have no solution or that have more than one possible solution. Some versions of these puzzles add a third type of inhabitant called a "Spy" who are allowed to make both true and false statements (we may add some spies in later). In this project, we will look at several puzzles of this type. You will be expected to both solve and justify your answers using sound logical reasoning.

Example: Suppose there are two inhabitants: Person A and Person B. A says: "We are both knaves". B says nothing.

Solution: A cannot be a knight, as a knight could not truthfully say that they are a knave. Then A must be a knave. Since A is a knave, their statement must be a lie (that is, is has to be false). In order for that to happen, B must not be a knave, so B must be a knight.

1. (1 point each) Either find the identity of each individual (knight or knave), or explain where there is more than one possible solution or no possible solution. Fully justify your solution.
   (a) A says: "B is a knave." B says: "We are both knights."
   (b) A says: "Either B is a knight or I am a knight." B says: "A is a knave."
   (c) A says: "We are either both knights or both knaves." B says: "A would tell you that I am a knave."
   (d) A says: "I am a knight or B is a knave." B says: "A is a knight and C is a knave." C says: "B and I are different types."
   (e) A says: "C is a knight." B says: "A and C are both knights." C says: "A is a knight or B is a knave."
   (f) A says: "B and I are both knights or both knaves." B says: "C and I are both knights." C says: "I could claim that A is a knave."
2. (1 point each) Suppose that we know that we have one individual of each type (knight, knave, or spy). Either find the identity of each individual , or explain where there is more than one possible solution or no possible solution. Fully justify your solution.
   (a) A says: "C is a knave." B says: "A is a knight." C says: "I am the spy."
   (b) A says: "I am not a spy." B says: "I am not a spy." C says: "I am not a spy."
   (c) A says: "I am a knight." B says: "I am not a spy" C says: "A is a spy."
   (d) A says: "C is a spy." B says: "C is not a knave." C says: "If you asked me, I would say that B is the spy."
3. Invent your own puzzle involving at least 3 individuals and at least two identity types. Then, provide full solution to your puzzle.