Math 311
Project 3 Handout

Instructions: This handout is designed to describe a project on one of the "Additional Topics" that may to choose to complete in order to satisfy the Project component of your grade in the course. As noted in the Course Policies Handout, you can earn 25 points (or more) based on the progress you make on these projects. To help make sure that you have time to work on these, you will be given time in class to work on these rather than our standard Daily Group Work assignments during the next three class periods (starting on Thursday, November 29th). You are encouraged to work in groups on these projects. You should turn in **three** projects for grading. One should be completed by Tuesday, December 4th. Your second is due on Thursday, December 6th, and your third project must be completed by Tuesday, December 11th. You will have the opportunity to earn up to 10-12 points on each of these projects. Any points beyond 25 points will count as extra credit.

Project 3: Single Pile 2-Player NIM Games

In $(1,2,3)-\ NIM$, the game begins with a pile of N stones. On their turn, a player can take either 1, 2, or 3 stones from the pile. The player that takes the last stone **wins** the game.

In $(1,2,3)-Misère\ NIM$, the game begins with a pile of N stones. On their turn, a player can take either 1, 2, or 3 stones. However, unlike in regular NIM, in this game, the player that takes the last stone **loses**.

Note that when analyzing games of this type, we assume players are trying to avoid losing and that they will make the most advantageous move possible when it is their turn to play.

1. (1 point) Which player has a winning strategy in $(1,2,3)-NIM$ if the game starts with 7 stones? Justify your answer.
2. (1 point) Which player has a winning strategy in $(1,2,3)-Misère\ NIM$ if the game starts with 7 stones? Justify your answer.
3. (2 points) Do a complete analysis of $(1,2,3)-\ NIM$. That is, determine which player has a winning strategy for any starting value of N [Hint: use backwards reasoning and split into cases].
4. (2 points) Do a complete analysis of $(1,2,3)-Misère\ NIM$. That is, determine which player has a winning strategy for any starting value of N.
5. (2 points) Generalize your previous result to the game $(1,2,3,...,k)-\ NIM$ in which a player can take from 1, 2, 3, ..., up to k stones.
6. (2 points) Do a complete analysis of $(2,3)-\ NIM$ in which a player can take either 2 or 3 stones. Note that is no-one is able to take the last stone, the game is considered a Draw.
7. (2 points) Invent and analyze your own NIM-like game.