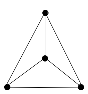
Math 311
Project 4 Handout

Instructions: This handout is designed to describe a project on one of the "Additional Topics" that may to choose to complete in order to satisfy the Project component of your grade in the course. As noted in the Course Policies Handout, you can earn 25 points (or more) based on the progress you make on these projects. To help make sure that you have time to work on these, you will be given time in class to work on these rather than our standard Daily Group Work assignments during the next three class periods (starting on Thursday, November 29th). You are encouraged to work in groups on these projects. You should turn in **three** projects for grading. One should be completed by Tuesday, December 4th. Your second is due on Thursday, December 6th, and your third project must be completed by Tuesday, December 11th. You will have the opportunity to earn up to 10-12 points on each of these projects. Any points beyond 25 points will count as extra credit.

Project 4: Vertex and Edge Patterns in Families of Graphs

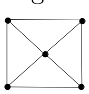
In this project, you will be asked to investigate patterns and find formulas for the number of edges, vertices, and the degrees of the vertices present in various infinite families of graphs.

1. (1 point) K_n denotes the **complete graph on** n **vertices**. In these simple graphs, there is exactly one edge between each distinct pair of vertices. For example, the figure below shows K_4, the complete graph on four vertices. Find a general formula in terms of n for the number of edges in K_n.



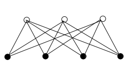
K_4

2. (1 point) W_n denotes the **wheel graph on** n **vertices**. In these simple graphs, there is "cycle" of n vertices and edges plus an additional vertex in the center, which is connected to each vertex in the external cycle by an edge. For example, the figure below shows W_4, the wheel graph on four vertices. Find a general formula in terms of n for the number of edges in W_n.



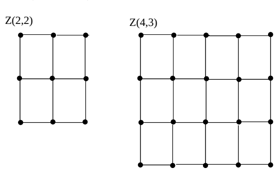
W_4

3. (1 point) $K_{m,n}$ denotes the **complete bipartite graph on** m and n **vertices**. In bipartite graphs, the vertices are divided into two subsets called **vertex classes**, and all edges connect a vertex from one class to a vertex in the opposite class. In a complete bipartite graph, every possible edge between vertices in different classes is present in the graph. For example, the figure below shows $K_{3,4}$. In order to help us keep track of which vertices are in each class, we often "color" the vertices in one class white and the vertices in the other class black. Find a general formula in terms of m and n for the number of edges in $K_{m,n}$.



$K_{3,4}$

4. $Z(m,n)$ denotes the graph enclosing a grid of $m \times n$ squares in the plane. For example:



   (a) (1 point) Find an expression for the number of vertices in $Z(m,n)$ in terms of m and n.
   (b) (1 point) Find an expression for the number of edges in $Z(m,n)$ in terms of m and n.
   (c) (1 point) Find an expression for the number of vertices v with $deg(v) = 2$.
   (d) (1 point) Find an expression for the number of vertices v with $deg(v) = 3$.
   (e) (1 point) Find an expression for the number of vertices v with $deg(v) = 4$.

5. Find (or invent) and clearly describe another infinite family of graphs. Find a formula for the number of edges in your graph in terms of the number of vertices present in the graph.