Math 311
Project Topic Descriptions Handout

Instructions: This handout is designed to give you brief descriptions of the "Additional Topics" that you have to choose to work on in order to satisfy the Project component of your grade in the course. As noted in the Course Policies Handout, you can earn 25 points (or more) based on the progress you make on these projects. To help make sure that you have time to work on these, you will be given time in class to work on these rather than our standard Daily Group Work assignments during the next three class periods (starting on Thursday, November 29th). You are encouraged to work in groups on these projects. I will be asking for your input on which projects sound most interesting to you, since I probably will not have time to put together project handouts for all of the topics described below.

1. **NIM Games:** In standard 2-player, 1-pile NIM games, the game begins with a single pile of stones. On their turn, a player can take some number of stones from the pile (different versions of the game put different restrictions on how many stones you are allowed to remove). The player that takes the last stone wins the game. This project will focus on developing effective strategies for winning the game under various sets of stone removal rules.
2. **Infinite Unions and Intersections of Sets:** This project revisits the idea of infinite unions and intersections of sets. You will be asked to explore how to represent different subsets of the real line using infinite unions and infinite intersections.
3. **Chicken Nugget Problems:** On your last take-home exam, we looked at one example of a "chicken nugget" problem. In particular, you were asked to find the biggest number of nuggets that you could not buy exactly when a specific collection of possible nuggets packs are for sold. In this project, you will be asked to look at a wider range of problems of this type.
4. **Well Orderings:** On Daily Group Work 23, we learned what it means for a poset to be well-ordered and discussed the question of whether $(\mathbb{N}, \leq)$ is well ordered. On this project, we will look more closely at this idea and what is known about finding orders that make various sets well-ordered.
5. **Tiling Problems:** Imagine using standard dominoes to cover squares of an $n \times n$ checkerboard. For which n is this possible? What if we looked at larger dominoes or more complicated tiles like "L" tiles or "T" tiles (think of playing Tetris). This project will investigate when it is possible to cover the board exactly with no overlaps, overhanging ends, and no missing squares.
6. **Vertices and Edges Patterns in Families of Graphs:** In this project, you will be asked to investigate patterns and find formulas for the number of edges, vertices, and the degrees of the vertices present in various infinite families of graphs.
7. **Planar Graphs:** In this project, you will investigate when it is possible to draw a graph in the plane without any edges crossing and when it is not possible to do so. This includes the famous "three utility lines" problem.
8. **Graph Vertex Colorings and Map Colorings:** This project will begin by investigating ways to color the vertices of a graph in such a way that no adjacent vertices share a color. This naturally connects with the problem of assigning colors to the regions of a graph and the famous "4-color" Theorem.
9. **Finite State Automata and Formal Languages:** This project will investigate using labeled directed graphs as a means of defining "accepted languages" by defining legal words as those that label paths in the graph that lead to and from specified vertices in the graph.
10. **Infinite Sets and Cardinality** This project will look at the cardinality of infinite sets. In particular, you will be asked to address the question of whether or not all infinite sets are the same size, or if there are different sized "infinities".
11. **Knight and Knave Logic Puzzles:** Logician and philosopher Raymond Smullyan invented a series of logic puzzles based on "knights", who always tell the truth and "knaves" who always lie (and perhaps some other more ambiguous characters). This project will investigate when and how one can determine the identities of individuals in such scenarios based on the statements they make.
12. **The Cantor Set of Other Fractals:** This project will investigate properties the Cantor Set, the Sierpinski triangle, and other 1D And 2D fractal figures.

Note: We will use AMS "dot voting" to help determine which projects I develop and bring for you to start working on in class this Thursday.