Math 450

Exam 1 Review Sheet

Section 1.1: Review of Calculus

• Know and be able to apply the definitions of a limit, continuity, the derivative, and the Riemann integral.
• Know the statements of (and be able to apply) the Intermediate Value Theorem, Rolle's Theorem, the Mean Value Theorem, and the Extreme Value Theorem.
• Be able to construct the Taylor Series for given function. Also know how to use Taylor's Theorem to compute an upper bound for the error when using a Taylor Polynomial to approximate the value of a function.

Section 1.2: Round-off Errors and Computer Arithmetic

• Understand the idea of computational error and how it arises within a computational device.
• Understand and be able to carry out rounding, chopping, and k-digit arithmetic.
• Understand and be able to compute the absolute error and the relative error of an approximation.
• Understand the error bound necessary for an approximation to be good to within d significant digits.
• Understand that changing the form of an expression can impact the computational error when evaluating the expression.
• Be able to put a polynomial into nested form. Also be able to compare the computational effectiveness of equivalent forms of an expression.
• Be able to use algebra to demonstrate that two different expressions are equivalent or to derive a new expressions that is equivalent to a given expression.

Section 1.3: Algorithms and Convergence

• Understand the definition of an algorithm and be able to interpret and produce pseudocode for a simple algorithm.
• Understand what it means for an algorithm to be stable, conditionally stable, and unstable.
• Understand the difference between linear and exponential error growth.
• Understand the definition of the rate of convergence of a limit or sequence. Also be able to find the rate of convergence for a given limit or sequence.

Section 2.1: The Bisection Method

• Know the Algorithm for the Bisection Method, the hypotheses necessary in order to apply this algorithm, and be able to apply it to find a root of a function.
• Understand the proof that the Bisection Method converges and know the order of convergence for this algorithm.
• Be able to find an upper bound on the number of iterations necessary in order to find an approximation of a root to within a given error tolerance.

Section 2.2: Fixed Point Iteration

• Know the Algorithm for Fixed Point Iteration, the hypotheses necessary in order to apply this algorithm, and be able to apply it to find a root of a function.
• Understand the proof that Fixed Point Iteration converges and know the order of convergence for this algorithm.
• Be able to find an upper bound on the number of iterations necessary in order to find an approximation of a root to within a given error tolerance.
• Given a function $f(x)$ that you want to find a root for, be able to find related function(s) to which Fixed Point iteration can be applied.
• Be able to determine whether a function on a given interval satisfies the hypotheses for the Fixed Point Iteration Algorithm. Also be able to find an interval that satisfies the hypotheses, and be able to find a constant k such that $|g'(x)| \leq k < 1$ on the interval.

Section 2.3: Newton's Method

• Know the Algorithm for Newton's Method, the hypotheses necessary in order to apply this algorithm, and be able to apply it to find a root of a function.
• Understand the proof that Newton's Method converges.
• Know the Algorithm for Secant Method, the hypotheses necessary in order to apply this algorithm, and be able to apply it to find a root of a function.
• Know the Algorithm for the Method of False Position, the hypotheses necessary in order to apply this algorithm, and be able to apply it to find a root of a function.
• Be able to compare the computational effectiveness of these three algorithms when used to approximate a root of a specific function.