Math 450
Exam 2 Review Sheet

Section 2.4: Error Analysis for Iterative Methods

• Know the definition of convergence of a sequence $\{p_n\}_{n=0}^{\infty} \to p$ of **order** α with **asymptotic error constant** λ. Also be able to apply it to find the order of convergence for a specific sequence, or to determine whether or not a given sequence is *linearly convergent* or *quadratically convergent*.
• Know the definition of a **root of multiplicity** m of a function $f(x)$ and be able to find the multiplicity of a root p of a given function, and to express it in the form: $f(x) = (x-p)^m \cdot q(x)$.
• Understand the connection between a root p of multiplicity m and the value of derivatives of f evaluated at p as stated in Theorem 2.11
• Know and be able to apply the Modified Newton-Raphson Method to approximate a root of a function.

Section 2.5: Accelerating Convergence

• Know the definition of the **forward difference operator** Δ
• Understand the and be able to Apply Aitken's method and Steffensen's Method to accelerate the convergence of Newton's Method.

Section 2.6: Zeros of Polynomials and Muller's Method

• Know the definition of complex numbers and the basic properties of complex conjugates.
• Understand the definition of a polynomial, and know the statements of the Fundamental Theorem of Algebra, the Remainder Theorem, and the Factor Theorem.
• Know how to evaluate a polynomial function using synthetic division. Also be able to use synthetic division to find roots of a polynomial, to factor a polynomial, and to aid in carrying out Newton's Method.
• Be able to use Horner's method and deflation in order to find **all** real and complex roots of a polynomial function.
• Be able to use Muller's Method to find a root of a function. You do not need to memorize the formulas for a, b and c. I will provide these if I ask an "in-class" question on Muller's method.

Section 3.1 and 3.2: Interpolation and the Lagrange Polynomial

• Understand the definition of interpolation and know the statement of the Weierstrass Approximation Theorem.
• Know the definition of the Kronecker δ function.
• Know the definition of the Lagrange interpolating polynomial and be able to find the Lagrange interpolating polynomial to a function given the values of the function at $n+1$ distinct points.
• Know and be able to find the remainder term for a Lagrange interpolating polynomial and be able to use it to find an upper bound on the error of a particular approximation.
• Know the advantages and disadvantages of using Lagrange polynomials. In particular, remember that Lagrange polynomials lack permanence.
• Neville's Method will not be tested on the in-class exam, but may be on your next programming assignment.

Section 3.3 and 3.4: Divided Differences and Hermite Interpolation

• Know how to compute a divided difference table and be able to use it to find the Newton Divided Difference interpolating polynomial for a function given values of the function at $n+1$ distinct points.
• Know how to extend the method for computing Newton Divided Difference interpolating polynomials to cases where we know information about the derivatives of the original function at various points.
• Be able to extend the method for computing Newton Divided Difference interpolating polynomials to cases where the known values of the function are evenly spaced by computing a forward difference table and interpolating using $P_n(s)$.
• Know the definition of the backward difference operator ∇ and be able to extend the method for computing Newton Divided Difference interpolating polynomials to cases where the known values of the function are evenly spaced by computing a backward difference table and interpolating using $P_n(s)$.
• Know the definition of the Hermite osculating polynomial approximating a given function and its relationship with Lagrange interpolating polynomials.
• Be able to compute the Hermite interpolating polynomial approximating a given function and use it to approximate a function at a given point.