Math 450
Exam 1 – Take Home Portion – Due: Friday, September 21st by 12:00pm

Instructions:

- You will have until noon on Friday, September 21st to complete this exam.
- The credit given on each problem will be proportional to the amount of correct work shown. Answers without supporting work will receive little credit.
- Work your exam on separate sheets of paper. Be sure to number each problem and put your name on each page. Include printouts of any supporting graphs and computations.
- The only resources that are allowed for this exam are your textbook, your course notes, Maple, your calculator, and programs that you have written yourself (include a printout of your code and all output if you use a program to complete a portion of an assigned problem). If you use Maple, you **may not** use pre-programmed algorithms such as Newton's Method, solve, or fsolve. You **may** use functions, expressions, simplification, symbolic differentiation, and graphing features.
- You should not work collaboratively with your classmates. You should not consult with other individuals or make use of additional print or online resources. Doing so will be considered an act of academic dishonesty and will be dealt with accordingly.

1. Let $f(x) = \dfrac{xe^x}{e^x - 1} - 1.5$

   (a) (3 points) Prove that $f(x)$ has a root in $[0, 1]$. Call this root r.

   (b) (3 points) Use the Bisection Method to approximate r to 8 decimal places of accuracy.

   (c) (3 points) Find a related function $g(x)$ that could be used as a fixed point iteration formula to find r.

   (d) (5 points) Find an interval containing r for which the hypotheses of both parts of Theorem 2.3 are satisfied for the function $g(x)$ you found in part (c) [If you cannot, come up with different function that does work].

   (e) (3 points) Use the Newton's Method to approximate r to 8 decimal places of accuracy.

   (f) (5 points) Find an interval containing r for which the Newton recursion you used in part (e) satisfies the hypotheses of Theorem 2.3.

   (g) (3 points) Use the Secant Method to approximate r to 8 decimal places of accuracy.

   (h) (3 points) Use the Method of False position to approximate r to 8 decimal places of accuracy.

   (i) (3 points) Which of the methods you tried finds r most efficiently? Justify your answer.