Name:

1. Since  $\pi$  is irrational, it cannot be represented as the quotient of two integers. Despite this, the quantity  $\frac{22}{7}$  is sometimes used to represent  $\pi$ . How "good" an approximation of  $\pi$  is  $\frac{22}{7}$ ? Try to be as clear as possible about how you are measuring/defining "good".

- 2. Consider the function  $f(x) = e^x 3x$  and the related equation  $e^x 3x = 0$ . Our goal is to understand the solutions to this equation (and hence the zeroes of the function f(x)). Attach additional work if necessary.
  - (a) Prove that the equation  $e^x 3x = 0$  has at least one real solution. That is, there is at least one  $x \in \mathbb{R}$  such that f(x) = 0. [Hint: Do you need to actually find a specific value for x?]

(b) Determine the total number of real solutions to the equation  $e^x - 3x = 0$ . Justify your answer.

(c) Comment on whether or not you are able to find any **exact** solutions to this equation.

(d) For each solution that you are not able to find exactly, find a "good" approximation of the solution. Be as clear as you can about both how you found the approximation and how accurate your approximation is.

- 3. Using a Calculus textbook or reputable online sources, find and write out the following key definitions and theorems.
  - (a) Continuity of a function f(x) at x = a. (b) Differentiability of a function f(x) at x = a.

(c) Integrability of a function f(x) on an interval [a, b]. (d) The Intermediate Value Theorem

(e) The Extreme Value Theorem

(f) Rolle's Theorem

(g) The Mean Value Theorem

(h) Taylor's Theorem