Math 450
Programming Assignment 4
Due: Wednesday, December 12th

1. Write programs that implements each of the following numerical methods. Your algorithm should take as input a function $f(t, y)$, an interval $[a, b]$, an initial condition $y(a) = \alpha$, and a step size h. For the higher order Taylor Methods, you may perform the additional differentiations externally.

   (a) Euler's Method

   (b) Taylor's Method of order 2.

   (c) Taylor's Method of order 4.

   (d) The Midpoint Method.

   (e) Modified Euler's Method.

   (f) Heun's Method.

   (g) Runge-Kutta Order 4.

2. Given the initial value problem $y' = -ty + 4ty^{-1}$, $0 \leq t \leq 1$, $y(0) = 1$, use each of your algorithms to approximate the value of $y(1)$ using each of the algorithms you coded above:

   (a) For $h = 0.1$.

   (b) For $h = 0.01$

3. (a) Verify that the solution to this differential equation is: $y(t) = \sqrt{4 - 3e^{-t^2}}$

   (b) Find the absolute error of each of your estimates using $h = 0.1$.

   (c) Find the absolute error of each of your estimates using $h = 0.01$.

   (d) Compute the ratio of each pair absolute errors you found. What, if anything, do these ratios indicate about the rate of convergence of each numerical algorithm?