

**Part 1: Problem Solving, Estimation, and Set Theory**

**Key Topics:**

- Memorize the 7 problem solving strategies and be able to apply them to solve a specific problem.
- Know what a counterexample is and how to use one to disprove a mathematical statement.
- Understand rounding and compatible numbers, and know whether they lead to over-estimates or under-estimates
- Be able to express sets in both set-builder and roster notation, and be able to determine whether or not a set is well defined.
- Memorize the common numerical sets (whole, natural, integer, rational, and real numbers).
- Understand universal sets, the empty set, and sets of sets, the notation  $\in$ ,  $\notin$ , and  $n(A)$ .
- Understand Equality of sets vs. equivalence of sets, as well as subsets vs. proper subsets.
- Be able to count the subsets of a set directly and using Pascal's Triangle.
- Be able to find unions, intersections, differences, and compliments of sets, and know when two sets are disjoint.
- Understand Venn diagrams; be able to count the elements in the union of two sets.
- Be able to combine multiple set operations; know De Morgan's Laws for sets.
- Be able to illustrate combinations of set operations by shading a Venn diagram, and be able to name regions in a Venn diagram using set operations.
- Be able to count and organize survey information using sets and Venn diagrams.

**Part 2: Mathematical Logic**

**Key Topics:**

- Know the definitions of inductive and deductive reasoning, and be able to identify which of the two is being used in a given example.
- Be able to use inductive reasoning to make conjectures and deductive reasoning to do a simple proof.
- Know the definition of a statement, and the difference between simple and compound statements.
- Be able to determine whether or not an English sentence is logical statement.
- Know the symbols for and meanings of the 5 basic logical operations.
- Be able to translate from English statements to logical symbols and vice versa.
- Know the difference between universal and existential quantifiers.
- Be able to negate logical statements, including those with quantifiers.
- Memorize the truth tables for the 5 logical connectives.
- Know the difference between "inclusive or" and "exclusive or"
- Know the number of rows in the truth table of a logical expression with given number of variables and be able to build the truth table for a given logical expression.
- Know the definition of logical equivalence for two logical statements, and be able to use truth tables to determine whether or not two statements are logically equivalent.
- Know DeMorgan's Laws, and be able to use them to rewrite statements.
- Be able to identify the hypothesis and conclusion in a conditional statement.
- Given a conditional statement, be able to write the converse, inverse, and contrapositive of the statement, and know which of these are logically equivalent.
- Know the components of a logical argument: premises and a conclusion.
- Know how to translate an argument from English to symbolic form, and vice versa.
- Know what a tautology is, and be able to use a truth table to determine whether or not an argument is valid.
- Memorize the forms of the standard Valid arguments: the Law of Detachment, the Law of Contraposition, the Law of Syllogism, and Disjunctive Syllogism, and the Invalid arguments: the Fallacy of the Converse and the Fallacy of the Inverse.
- Be able to determine whether a given argument is valid by translating it into symbolic form and comparing it to the list of standard arguments.
- Know how to construct Euler diagrams for a quantified logical statement.
- Know how to analyze the validity of syllogism by drawing Euler diagrams

### Part 3: Counting and Probability

#### Key Topics:

- Understand Counting using tree diagrams, the Fundamental Counting Principle, and “Slot Diagrams”.
- Be able to tell whether a given example is a permutation, a combination, or neither.
- Memorize and be able to apply the counting formulas for permutations and combinations:  $P(n, r) = \frac{n!}{(n-r)!}$  and  $C(n, r) = \frac{n!}{r!(n-r)!}$
- Know how to use Pascal’s Triangle to count combinations.
- Understand how to count in situations involving cards, dice, coin flips, and simple lottery games.
- Know the definitions of: experiment, outcomes, sample space, and event, be able to describe an event as a subset of a sample space, and memorize the three basic properties of probability.
- Know the definition of the probability of an outcome and the probability of an event.
- Know the difference between Empirical Assignment of Probability and Theoretical Assignment of Probability.
- Be able to use counting to calculate the probability of an event and the “odds” of an event.
- Know how to write a given event as either the complement of an event or as the union of two other events, and memorize the formulas for computing the probability of the complement of an event and of the union of two events.
- Be able to apply Venn diagrams for probability to find the probability of events in a given situation.
- Understand conditional probability and be able to compute conditional probabilities both in the case that all outcomes are equally likely, and when outcomes are not all equally likely.
- Know the definition of independent and dependent events, and be able to apply it to a given pair of events.
- Be able to compute the probability of the intersection of two events, and know how to use a tree diagram to help compute probabilities of the intersection of multiple events.
- Understand expected value, and know how to use it to predict the “average” outcome of an experiment or game.
- Know how to use expected value to determine whether or not a given game is fair.

### Part 4: Visualizing Data and Descriptive Statistics

#### Key Topics:

- Know the definitions of population, data, and sample, and be able to tell which is which for a given example.
- Understand basic experimental design, and be able to identify different types of bias (selection bias, leading question bias, and non-response bias)
- Given a data set, be able to construct a frequency table, a relative frequency table, and both frequency and relative frequency histograms (bar graphs), and be able to interpret data given by a histogram.
- Be able to make stem and leaf displays of both one and two sets of data
- Understand and be able to compute the mean, median, mode, and midrange of a set of data listed singly, or given by a frequency table.
- Be able to find the 5 number summary of a data set and be able to draw its box-and-whisker plot.
- Understand which measures of center are effected by outliers, and be able to discuss which aspects of a data set are described by the different measures of center.
- Understand and be able to compute the range, standard deviation, and coefficient on variation for data that is listed as individual data points, or data that is given in a frequency table.
- Be able apply the coefficient of variation to compare the volatility of different data sets.
- Memorize the properties of a Normal Distribution, be able to draw the graph of a normally distributed population, indicating its mean and standard deviation.
- Be able to find areas under a normal curve using a  $z$ -table, and be able to compute the  $z$ -score of a raw score.
- Be able to work backwards from a  $z$ -score to a raw score, and from a percentage to a  $z$ -score.
- Be able to compare data and solve application problems by using  $z$ -scores.
- Be able to find the  $x$  and  $y$  intercepts of a linear equation, and be able to graph a linear equation.
- Be able to find the slope and  $y$ -intercept of a linear equation by putting it into slope-intercept form.
- Be able to find the slope of the line through a given pair of points, and understand effect of the slope of a linear equation on its graph.
- Be able to use a linear equation to model a real life situation.
- Know what a scatterplot is, and be able to sketch the line of best fit for a data set.
- Understand how the correlation coefficient describes whether two variables are positively or negatively correlated and how close a data set relating them is to being linear.