Math 143, Section 1
Exam 1 - Solutions **Name:**______________________________
01/31/2007

Instructions: You will have 50 minutes to complete this exam. Calculators are allowed, but this is a closed book, closed notes exam. I will give credit to each problem proportional to the amount of correct work shown. Correct answers without supporting work will receive little credit. Be sure to simplify answers when possible. Also, make sure to follow directions carefully on each problem.

1. Given the points $A : (-2, 5)$ and $B : (6, -3)$:

   (a) (3 points) Find the distance between A and B
   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-2))^2 + (-3 - 5)^2} = \sqrt{(8)^2 + (-8)^2}$
   $= \sqrt{64 + 64} = \sqrt{128} = 8\sqrt{2}$.

   (b) (3 points) Find the point C such that B is the midpoint of the line segment connecting A to C
   Let $C = (x, y)$ be the point we are looking for. Then, by the midpoint formula, $(\frac{x-2}{2}, \frac{y+5}{2}) = B = (6, -3)$. That is, $\frac{x-2}{2} = 6$, whereby $x - 2 = 12$, so $x = 14$, and $\frac{y+5}{2} = -3$, therefore $y + 5 = -6$, so $y = -11$. Thus $C = (14, -11)$.

   (c) (3 points) Find an equation for the line containing A and B
   First, we find the slope: $m = \frac{y_2 - y_1}{x_2 - x_2} = \frac{-3-5}{6-(-2)} = \frac{-8}{8} = -1$.
   Then, by point slope, $y - 5 = -1(x + 2) = -x - 2$, so $y = -x + 3$.

   (d) (3 points) Find an equation for the vertical line containing A
   Recall that vertical lines have fized x-coordinates, and y is allowed to vary.
   Thus the equation for the vertical line containing A is $x = -2$

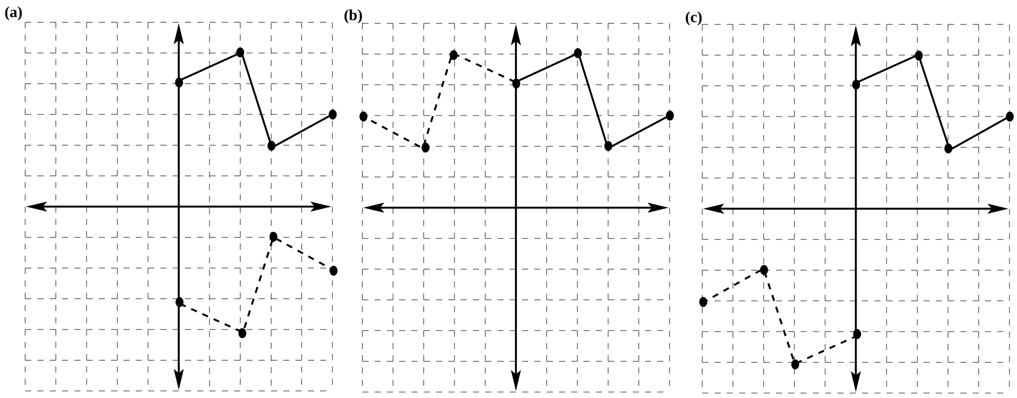
   (e) (3 points) Find an equation for a line that is perpendicular to the line containing A and B
   Recall that two lines are perpendicular if and only if their slopes are negative reciprocals of each other. Therefore, any line with slope $m' = -\frac{-1}{1} = 1$ is perpendicular to the line containing A and B. That is, any line of the form $y = x + b$. For example, $y = x$ will do nicely.

2. (3 points each) Sketch the remaining part of the graph if the given symmetry property is true:

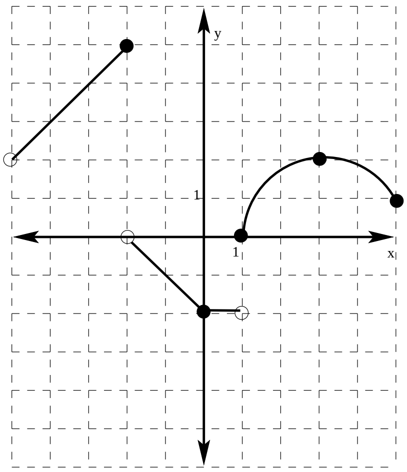
   (a) The graph is symmetric with respect to the x-axis.

   (b) The graph is symmetric with respect to the y-axis.

   (c) The graph is symmetric with respect to the origin.



3. (2 points each) For the given graph of $f(x)$, find the following:



(a) $f(0)$ and $f(3)$

From the graph, $f(0) = -2$ and $f(3) = 2$

(b) x, if $f(x) = 1$

$f(x) = 1$ when $x \approx 1.25$, and when $x = 5$.

(c) The domain of f

Domain: $(-5, 5]$

(d) The range of f

Range: $[-2, 5]$

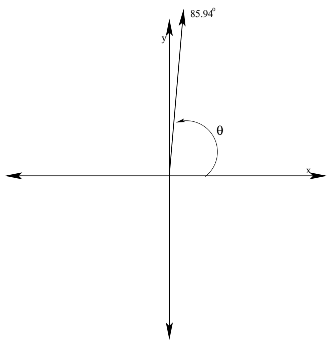
(e) The intervals where f is increasing.

$(-5, -2) \cup [1, 3]$

4. (3 points each) Given the angle $\theta = 1.5$ radians

(a) Express θ in terms of degrees, with your answer rounded to the nearest hundredth of a degree.
1.5 radians $\cdot \frac{180°}{\pi} \approx 85.94°$

(b) Draw θ in standard position



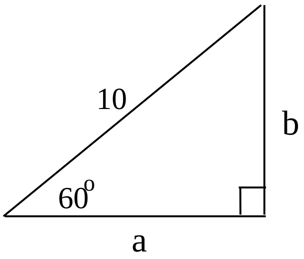
(c) Convert θ into degree, minute, second form.
$85.94° = 85° + .94(60)' = 85°56.4' = 85°56'.4(60)'' = 85°56'24''$ (or, if you decided not to round before converting, $85.94° = 85°56'37''$)

(d) Find three angles (in degrees) that are coterminal with θ
The idea here is to add or subtract multiples of 360°.
Acceptable answers include: $-994.06°, -634.06°, -274.06°, 445.94°, 805.94°$, and $1165.94°$.

5. (6 points) Bob has a unicycle whose wheel is two feet in diameter. Suppose that he rides it for a mile. How many revolutions does the wheel complete during the ride? (Recall: 1 mile = 5280 feet)

Notice that since the diameter of the wheel is 2 feet, the radius is 1 foot. Thus the circumference is 2π feet. Therefore, to find the number of revolutions, we take $\frac{5280/,/,\text{feet}}{2\pi \text{ feet per rev}} \approx 840.34$ revolutions

6. (6 points) Find the values of a and b *exactly* based on the triangle below:



By definition, $\cos 60° = \frac{a}{10}$, so $a = 10\cos 60°$. But, recall that $\cos 60° = \frac{1}{2}$.
Hence $a = (10)\frac{1}{2} = 5$.

Similarly, $b = 10\sin 60°$, and $\sin 60° = \frac{\sqrt{3}}{2}$, so $b = (10)\frac{\sqrt{3}}{2} = 5\sqrt{3}$

(We could also have found b using the Pythagorian Theorem).

7. (5 points each) Verify the following identities by transforming the left hand side into the right hand side:

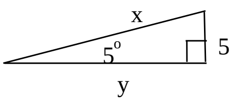
   (a) $(\sin\theta + \cos\theta)^2 = 2\sin\theta\cos\theta + 1$
   $(\sin\theta + \cos\theta)^2 = (\sin\theta + \cos\theta)(\sin\theta + \cos\theta) = \sin^2\theta + \sin\theta\cos\theta + \cos\theta\sin\theta + \cos^2\theta$
   $= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\sin\theta\cos\theta + 1$ (Since $\sin^2\theta + \cos^2\theta = 1$)

   (b) $\sin\theta + \cos\theta\cot\theta = \csc\theta$
   $\sin\theta + \cos\theta\cot\theta = \sin\theta + \cos\theta\frac{\cos\theta}{\sin\theta} = \sin\theta + \frac{\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta+\cos^2\theta}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$

8. (7 points) Fill in *exact* values in each blank in the table below:

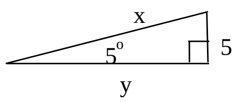
θ (degrees)	θ (radians)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	undefined
180°	π	0	-1	0
270°	$\frac{3\pi}{2}$	-1	0	undefined

9. (7 points) Suppose you are constructing an accessibility ramp that needs to reach a height of 5 feet. What is the minimum length of the ramp if safety regulations require that the grade of the ramp is no steeper than 5°?



From the triangle above which visualizes the situation described in this problem, we see that $\sin 5° = \frac{5}{x}$, where x is the minimum length the the ramp, in feet. Then $x = \frac{5}{\sin 5°} \approx 57.37$ feet.

10. (6 points) Given that $\sin\theta = \frac{6}{11}$ and $\cos\theta < 0$, find the exact value of both $\cos\theta$ and $\tan\theta$.

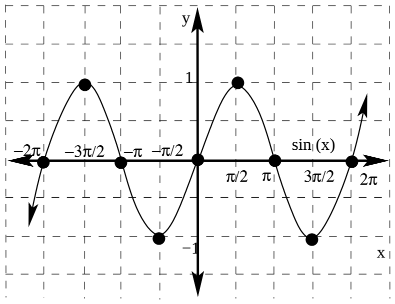


Using the triangle above, $x = 11^2 - 6^2 = 121 - 36 = 85$. Also, since $\sin\theta > 0$ and $\cos\theta < 0$, θ is in the second quadrant, so $\tan\theta < 0$

Therefore, $\cos\theta = -\frac{\sqrt{85}}{11}$ and $\tan\theta = -\frac{6}{\sqrt{85}}$

11. (6 points each) In the space below, carefully draw the graphs of $f(t) = \sin t$ and $f(t) = \csc t$ for $-2\pi \leq t \leq 2\pi$

(a) $f(t) = \sin t$



(b) $f(t) = \csc t$