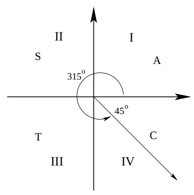
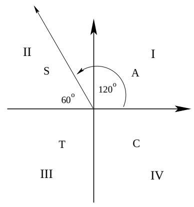
Math 143
Exam 2 - Solutions **Name:**______________________________
02/21/2007

Instructions: You will have 50 minutes to complete this exam. Calculators are allowed, but this is a closed book, closed notes exam. I will give credit to each problem proportional to the amount of correct work shown. Correct answers without supporting work will receive little credit. Be sure to simplify answers when possible. Also, make sure to follow directions carefully on each problem.

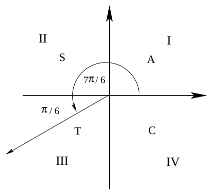
1. (3 points each) Find the exact value of the following:



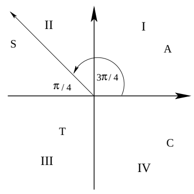
   (a) $\sin(315°) = -\sin(45°) = -\frac{\sqrt{2}}{2}$



   (b) $\tan(120°) = -\tan(60°) = -\frac{\sin(60°)}{\cos(60°)} = -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$



   (c) $\cos(\frac{7\pi}{6}) = -\cos(\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$



   (d) $\sec(\frac{3\pi}{4}) = -\sec(\frac{\pi}{4}) = -\frac{1}{\cos(\frac{\pi}{4})} = -\frac{1}{\frac{\sqrt{2}}{2}} = -\sqrt{2}$

2. (5 points each) Find all solutions to the following equations with $0 \leq \theta \leq 360°$. Approimate your solutions to within .01°

   (a) $\cos\theta = .4$

   To solve this, we use inverse trigonometry functions. $\cos^{-1}(.4) \approx 66.42°$
   This gives one possible solution, but remember there is a second solution, which we find by subtracting from 360°, due to the symmetry and periodic properties of cos. Thus $360 - 66.42 = 293.58°$ is also a solution between 0° and 360°.

   (b) $\tan\theta = 5$

   To solve this, we use again use inverse trigonometry functions. $\tan^{-1}(5) \approx 78.69°$
   Once again, there is a second solution, which we find by adding 180°, due to the periodic properties of tan. Thus $78.69 + 180 = 258.69°$ is also a solution between 0° and 360°.

3. (3 points each) Determine whether the following function are even, odd, or neither:

(a) $f(x) = 4 + 5x - x^7$

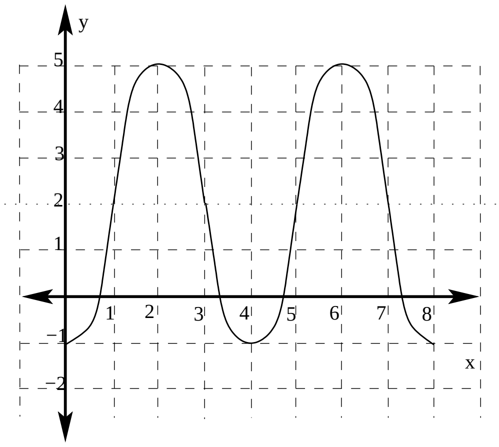
$f(-x) = 4 + 5(-x) - (-x)^7 = 4 - 5x + x^7$, while $-f(x) = -4 - 5x + x^7$.
Therefore, $f(x)$ is neither even nor odd.

(b) $g(x) = x^3 - \sin x$

$g(-x) = (-x)^3 - \sin(-x) = -x^3 + \sin x$, while $-g(x) = -x^3 + \sin x$.
Therefore, $g(x)$ is odd.

(c) $h(x) = 4\cos x + 2$

$h(-x) = 4\cos(-x) + 2 = 4\cos x + 2 = h(x)$.
Therefore, $f(x)$ is even.



4. Given the graph:

(a) (4 points) Find the amplitude, period, and midline for the graph.

From the graph above, we see that the max is 5, and the min is -1, so the amplitude is half of this range, or 3. Also, the period of the graph is 4, and the midline is $y = 2$.

(b) (4 points) Express the function shown with an equation of the form: $y = a\sin(bt + c) + d$

Since the period is 4, we know that $\frac{2\pi}{b} = 4$, or $4b = 2\pi$, thus $b = \frac{\pi}{2}$
Since the midline is $y = 2$, $d = 2$, and since the amplitude is 3, $a = 3$.
Finally, if this is a sine graph, it has been shifted 2 units right.
Therefore, we have the equation: $y = 3\sin(\frac{\pi}{2}(t-1)) + 2$, or $y = 3\sin(\frac{\pi}{2}t - \frac{\pi}{2}) + 2$

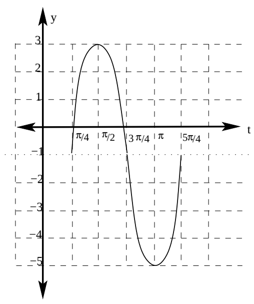
(c) (4 points) Express the function shown with an equation of the form: $y = a\cos(bt + c) + d$

For the cosine version of this graph, everything remains the same as above except the shift is now 2 units right. Therefore, we have the equation: $y = 3\cos(\frac{\pi}{2}(t-2)) + 2$, or $y = 3\cos(\frac{\pi}{2}t - \pi) + 2$.
Note: it is also acceptable to write this using a vertical reflection and no horizontal shift, which gives the equation $y = -3\cos(\frac{\pi}{2}t) + 2$.

5. For each function below, find the amplitude and period of the function, and then carefully draw the graph the the function.

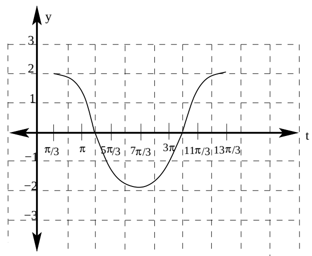
(a) (6 points) $y = 4\sin(2t - \frac{\pi}{2}) - 1$

Notice that the amplitude is 4, and since $b = 2$, the period is $\frac{2\pi}{2} = \pi$. We also see that the midline is $y = -1$, and the phase shift is $\frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$. We can also see where the graph begins and ends, and doublecheck the period by solving the inequality: $0 \leq 2t - \frac{\pi}{2} \leq 2\pi$, which gives $\frac{\pi}{2} \leq 2\pi \leq \frac{5\pi}{2}$, or $\frac{\pi}{4} \leq t \leq \frac{5\pi}{4}$. The result is the graph below:



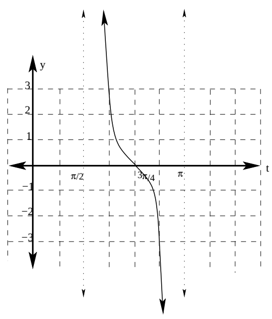
(b) (6 points) $y = 2\cos(\frac{1}{2}t - \frac{\pi}{6})$

Notice that the amplitude is 2, and since $b = \frac{1}{2}$, the period is $\frac{2\pi}{\frac{1}{2}} = 4\pi$. We also see that the midline is $y = 0$, and the phase shift is $\frac{\frac{\pi}{6}}{\frac{1}{2}} = \frac{\pi}{3}$. We can see where the graph begins and ends, and doublecheck the period by solving the inequality: $0 \leq \frac{1}{2}t - \frac{\pi}{6} \leq 2\pi$, which gives $\frac{\pi}{6} \leq \frac{1}{2}t \leq \frac{13\pi}{6}$, or $\frac{\pi}{3} \leq t \leq \frac{13\pi}{3}$. The result is the graph below:



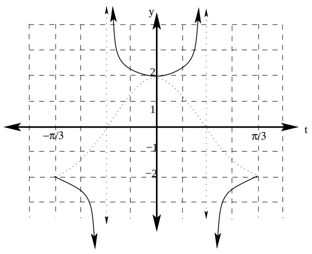
(c) (6 points) $y = 3\cot(2t - \pi)$

Notice that the amplitude is 3, and since $b = 2$, and the standard period of cot is π, the period is $\frac{\pi}{2}$. We also see that the midline is $y = 0$, and the phase shift is $\frac{\pi}{2}$. Since a standrad cot graph sits between 0 and π, we can see where the graph begins and ends, and doublecheck the period by solving the inequality: $0 \leq 2t - \pi \leq \pi$, which gives $\pi \leq 2t \leq 2\pi$, or $\frac{\pi}{2} \leq t \leq \pi$. The result is the graph below:



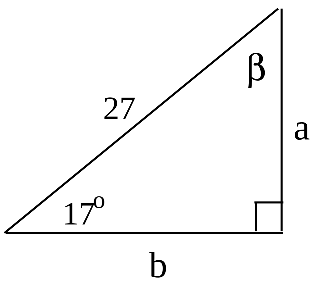
(d) (6 points) $y = -2\sec(3t + \pi)$

We will graph this by irst considering the related cosine graph, $y = -2\cos(3t + \pi)$. Notice that the amplitude is 2, and there is a reflection across the midline. Since $b = 3$, the period is $\frac{2\pi}{3}$. We also see that the midline is $y = 0$, and the phase shift is $-\frac{\pi}{3}$. We can see where the graph begins and ends, and doublecheck the period by solving the inequality: $0 \leq 3t + \pi \leq 2\pi$, which gives $-\pi \leq 3t \leq \pi$, or $-\frac{\pi}{3} \leq t \leq \frac{\pi}{3}$. The result is the graph below:

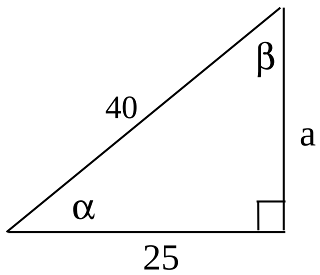


6. (6 points each) Given the indicated parts of the triangle $\triangle ABC$, find all remaining parts. Estimate your answers to within 2 decimal places.

(a)



(b)

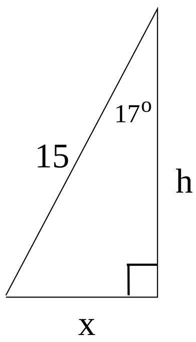


For part (a), First, since the angles of a triangle total 180°, $\beta = 90° - \alpha = 90° - 17° = 73°$. Next, notice that $\sin 17° = \frac{a}{27}$, so $a = 27\sin 17° \approx 7.89$. Similarly, $\cos 17° = \frac{b}{27}$, so $b = 27\cos 17° \approx 25.82$.

For part (b), by the Pythagorean Thoerem, $40^2 = a^2 + 25^2$, so $a = \sqrt{40^2 - 25^2} \approx 31.22$. Next, $\cos\alpha = \frac{25}{40}$, therefore, $\alpha = \cos^{-1}(\frac{25}{40}) \approx 51.32°$. Finally, $\beta = 90 - \alpha \approx 90° - 51.32° = 38.68°$

7. (12 points) A 15 foot extension ladder is leaning against a wall forming an angle of 17°.

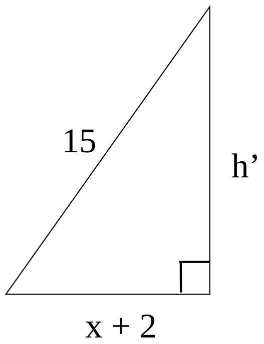
(a) How far is the base of the ladder from the wall?



Based on the triangle constructed above, $\sin 17° = \frac{x}{15}$. Therefore, $x = 15\sin 17° \approx 4.39$ feet.

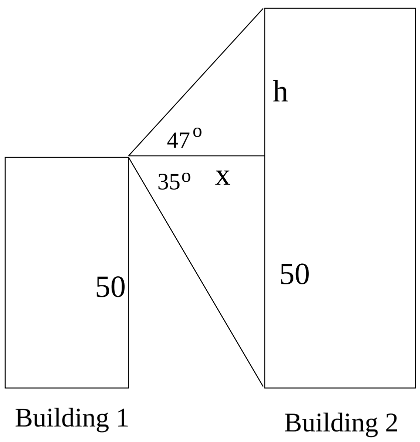
(b) If the distance between the base of the ladder and the wall is increased by 2 feet, how much does the top of the ladder move down the wall?

First notice that based on the triangle above, since $\cos 17° = \frac{h}{15}$, the original height of the ladder was $h = 15 \cos 17° \approx 14.34$. To find the new height, we build a triangle for the ladder after its base has been moved 2 feet further from the wall:



Based on the new triangle, using the Pythagorean Theorem, we see that $h' = \sqrt{15^2 - (x+2)^2} = \sqrt{15^2 - (6.39)^2} \approx 13.57$ feet. Therefore, the top of the ladder moved aproximately $14.34 - 13.57 = .77$ feet down the wall.

8. (12 points) Suppose you are standing on the roof of a 50 foot tall building. From your position on the roof, you have a clear view of a taller building across the street. You measure the angle of elevation to the top of the other building and find that it is 47°, while the angle of depression to the base of the other building is 35°. Find the height of the second building.



The diagram above shows the situation described in this problem. To find the total height of the second building, we first find the distance between the two buildings using the fact that $\tan 35° = \frac{x}{50}$, therefore, $x = \frac{50}{\tan 35°} \approx 71.41$ feet.

Next, we notice that $\tan 47° = \frac{h}{x}$, so $h \approx 71.41 \tan 47° \approx 76.6$ feet.

Thus, the total height of the building is approximately $50 + 76.6 = 126.6$ feet.