Math 143
Final Exam Practice Problems - Part 2

1. Find the *exact* value of the following:

   (a) $\sin(\frac{-\pi}{12})$
   (b) $\cos(105^\circ)$

2. Given that $\csc\theta = -\frac{5}{4}$ and $\cos\phi = \frac{12}{13}$, where θ is in the third quadrant, and ϕ is in the fourth quadrant, find *exact* values of:

   (a) $\cos\theta$
   (b) $\sin(2\phi)$
   (c) $\cos(\phi - \theta)$
   (d) $\sin(\theta + \phi)$

3. Verify the following identities by transforming the left hand side into the right hand side:

   (a) $\cos\left(\dfrac{3\pi}{2} - \theta\right) = -\sin\theta$
   (b) $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x \sec x$
   (c) $\sec^2 t - \csc^2 t = \dfrac{\tan t - \cot t}{\sin t \cos t}$
   (d) $\sin 3t \cos 3t = \dfrac{1}{2}\sin 6t$
   (e) $\dfrac{1+\cos 2t}{\sin 2t} = \cot t$

4. Find *exact* solutions to the following equations with $0 \le \theta < 2\pi$.

   (a) $2\sin(4x + \frac{\pi}{4}) = -\sqrt{3}$
   (b) $4\cos^3\theta = 3\cos\theta$
   (c) $\sin 2t - \sin t = 0$
   (d) $2\cos^2\theta - 5\cos\theta - 5 = 0$

5. Given the tables below, find the following:

x	0	2	4	6	8
f(x)	1	5	8	4	0

x	0	2	4	6	8
g(x)	2	6	5	9	7

   (a) $f^{-1}(5)$
   (b) $f(g^{-1}(9))$
   (c) $g(f^{-1}(4))$

6. Determine whether or not the following functions are one-to-one. You must justify your answer to each part.

   (a) $f(x) = 3x^2 - 2$
   (b) $g(x) = \dfrac{4}{x}$

7. Find the *exact* value of the following:

   (a) $\sin^{-1}(-\frac{\sqrt{3}}{2})$

   (b) $\tan^{-1}(-\sqrt{3})$

   (c) $\cos^{-1}(-\pi)$

   (d) $\cos(\cos^{-1}(-\frac{1}{2}))$

   (e) $\sin^{-1}(\sin(\frac{2\pi}{3})$

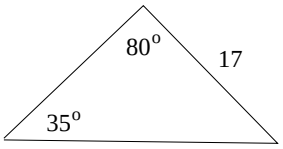
   (f) $\tan(\cos^{-1}(\frac{1}{2}))$

   (g) $\cos(2\tan^{-1}(-\frac{5}{7}))$

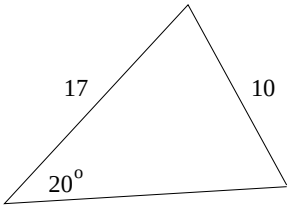
8. Express $\tan(\cos^{-1}(\frac{x}{x^2-1}))$ algebraically.

9. Solve the following triangles: (these are not neccessarily drawn to scale)

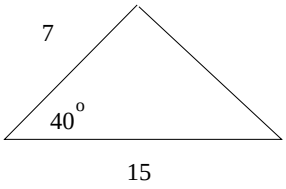
(a)



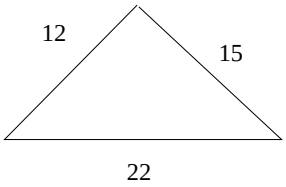
(b)



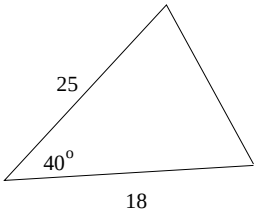
(c)

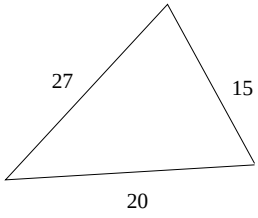


(d)

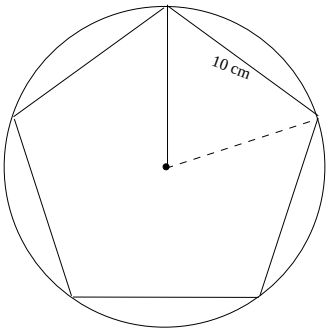


10. Find the area of the following triangles

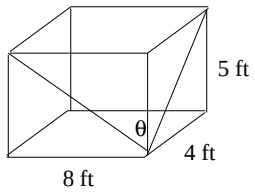




11. Suppose that a regular pentagon inscribed in a circle has sides of length 10cm. Find the area of the pentagon (See the figure below).



12. A rectangular box measures 8 feet by 4 feet by 5 feet. Find the angle θ between the diagonal on the front of the box with the diagonal on one of the sides of the box. (See the figure below).



13. Express the following in the form $a + bi$. You do **not** have to use trigonometric forms.

    (a) $(7 - 2i) - (6 + 11i)$

    (b) $(7 - 2i)(6 + 11i)$

    (c) $\dfrac{7 - 2i}{6 + 11i}$

    (d) i^{23456}

14. Let $z_1 = -5 - 5i$ and $z_2 = -12 + 5i$

    (a) Find the trigonometric form of z_1

    (b) Find the trigonometric form of z_2

    (c) Express $(z_1)^6$ in the form $a + bi$

    (d) Find the fourth roots of $z_1 = -5 - 5i$

15. Change the following from polar coordinates to rectangular coordinates:

    (a) $(-3, \pi)$

    (b) $(4, \frac{5\pi}{3})$

    (c) $(-3, \frac{17\pi}{3})$

16. Change the following from rectangular coordinates to polar coordinates:

    (a) $(3, -3)$

    (b) $(2\sqrt{3}, -2)$

    (c) $(-7, -1)$

17. Write the following equations in polar coordinates:

    (a) $4y = x$

    (b) $3y - 4y = 12$

18. Graph the following polar equations:

    (a) $r = 3\sin 2\theta$

    (b) $r = 2\sin 3\theta$

    (c) $r = 2 - 2\sin\theta$