Math 143
Polar Coordinates Problems

1. Change the following from polar coordinates to rectangular coordinates:

   (a) $(5, \frac{\pi}{3})$
   Recall that $x = r\cos\theta$ and $y = r\sin\theta$
   Therefore, $x = 5\cos(\frac{\pi}{3}) = 5(\frac{1}{2}) = \frac{5}{2}$.
   Similarly, $y == 5\sin(\frac{\pi}{3}) = 5(\frac{\sqrt{3}}{2}) = \frac{5\sqrt{3}}{2}$.
   Thus, the rectangular coordinate of this point is: $(\frac{5}{2}, \frac{5\sqrt{3}}{2})$.

   (b) $(7, \frac{11\pi}{6})$
   Recall that $x = r\cos\theta$ and $y = r\sin\theta$
   Therefore, $x = 7\cos(\frac{11\pi}{6}) = 7(\frac{\sqrt{3}}{2}) = \frac{7\sqrt{3}}{2}$.
   Similarly, $y == 7\sin(\frac{11\pi}{6}) = 7(-\frac{1}{2}) = -\frac{7}{2}$.
   Thus, the rectangular coordinate of this point is: $(\frac{7\sqrt{3}}{2}, -\frac{7}{2})$.

   (c) $(-4, \frac{3\pi}{2})$
   Note: This problem is probably easier to do just by plotting this point and using common sense, but formally, recall that $x = r\cos\theta$ and $y = r\sin\theta$.
   Therefore, $x = -4\cos(\frac{3\pi}{2}) = -4(0) = 0$.
   Similarly, $y == -4\sin(\frac{3\pi}{2}) = -4(-1) = 4$.
   Thus, the rectangular coordinate of this point is: $(0, 4)$.

2. Change the following from rectangular coordinates to polar coordinates:

   (a) $(-5, 0)$
   Here, we could one again just use common sense to find the polar coordinates. Formally, we can use the fact that $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$.
   Therefore, $r = \sqrt{(-5)^2 + 0^2} = \sqrt{25} = 5$, and $\theta = \pi$. Notice that blindly taking $\tan^{-1}(\frac{0}{-5})$ would give us $\theta = 0$, which is incorrect, since it is in the wrong quadrant.
   Thus, the coordinates of this point in polar coordiantes are: $(5, \pi)$.

   (b) $(2, 2\sqrt{3})$
   Here, $r = \sqrt{x^2 + y^2} = \sqrt{(2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$.
   Also, $\tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$, so $\theta_R = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$, which, since this is in the correct quadrant (quadrant 1), gives $\theta = \frac{\pi}{3}$.
   Therefore, the coordinates of this point in polar coordiantes are: $(4, \frac{\pi}{3})$.

   (c) $(5, -7)$
   Here, $r = \sqrt{x^2 + y^2} = \sqrt{(5^2 + (-7)^2} = \sqrt{25 + 49} = \sqrt{74}$.
   Also, $\tan\theta = -\frac{7}{5}$, so $\theta_R = \tan^{-1}(-\frac{7}{5})$, which, since this is in the correct quadrant (quadrant 2), gives $\theta = \tan^{-1}(-\frac{7}{5})$.
   Therefore, the coordinates of this point in polar coordiantes are: $(\sqrt{74}, \tan^{-1}(-\frac{7}{5}))$.

3. Write the following equations in polar coordinates:

   (a) $x^2 + y^2 = 49$
   Since $r^2 = x^2 + y^2$, substituting this equation becomes $r^2 = 49$, or $r = 7$.

(b) $y = -3x$

Recall that $x = r\cos\theta$ and $y = r\sin\theta$. Therefore, substituting, $r\sin\theta = -3r\cos\theta$, or $\sin\theta = -3\cos\theta$. Then $\frac{\sin\theta}{\cos\theta} = -3$, or $\tan\theta = -3$. Thus, the equation becomes:
$\theta = \tan^{-1}(-3)$

(c) $y = x - 5$

Again using the fact that $x = r\cos\theta$ and $y = r\sin\theta$, we substitute in order to obtain $r\sin\theta = r\cos\theta - 5$, or $r\sin\theta - r\cos\theta = -5$.
Then $r(\sin\theta - \cos\theta) = -5$, or $r = \frac{-5}{\sin\theta - \cos\theta} = \frac{5}{\cos\theta - \sin\theta}$ is the simplified polar form of this equation

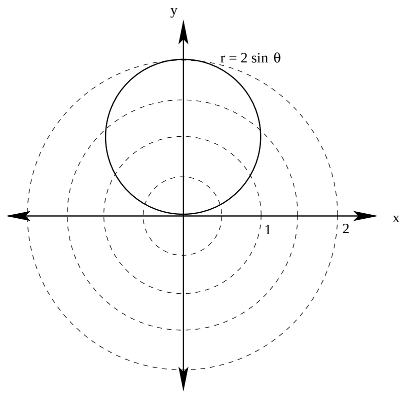
(d) $(x-1)^2 + y^2 = 1$

Again using the fact that $x = r\cos\theta$ and $y = r\sin\theta$, we substitute in order to obtain $(r\cos\theta - 1)^2 + (r\sin\theta)^2 = 1$, or $r^2\cos^2\theta - 2r\cos\theta + 1 + r^2\sin^2\theta = 1$.
Then $r^2(\cos^2\theta + \sin^2\theta) - 2r\cos\theta = 0$, or $r^2 = 2r\cos\theta$
Hence $r = 2\cos theta$ is the simplified polar form of this equation.

4. Graph the following polar equations:

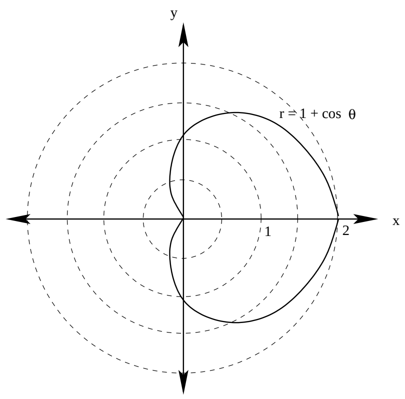
(a) $r = 2\sin\theta$

θ	$\sin\theta$	$r = 2\sin\theta$
0	0	0
$\frac{\pi}{6}$	$\frac{1}{2}$	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	1.414
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	1.7
$\frac{\pi}{2}$	1	2
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	1.7
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	1.414
$\frac{5\pi}{6}$	$\frac{1}{2}$	1
π	0	0

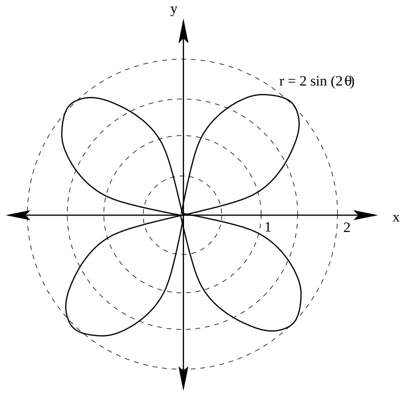


(b) $r = 1 + \cos\theta$

θ	$\cos\theta$	$r = 1 + \cos\theta$
0	1	2
$\frac{\pi}{6}$	$\frac{\sqrt{2}}{2}$	1.86
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	1.7
$\frac{\pi}{3}$	$\frac{1}{2}$	1.5
$\frac{\pi}{2}$	0	1
$\frac{2\pi}{3}$	$-\frac{1}{2}$	.5
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	.3
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	.14
π	-1	0
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	.14
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	.3
$\frac{4\pi}{3}$	$-\frac{1}{2}$	.5
$\frac{3\pi}{2}$	0	1
$\frac{5\pi}{3}$	$\frac{1}{2}$	1.5
$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	1.7
$\frac{11\pi}{6}$	$\frac{\sqrt{2}}{2}$	1.86
2π	1	2



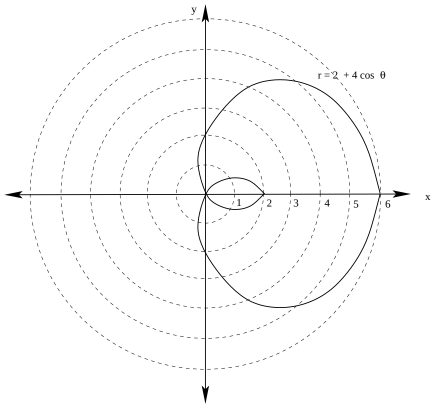
(c) $r = 2\sin(2\theta)$



θ	2θ	$\sin 2\theta$	$r = 2\sin(2\theta)$
0	0	0	0
$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{1}{2}$	1
$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	1.7
$\frac{\pi}{4}$	$\frac{\pi}{2}$	1	2
$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	1.7
$\frac{5\pi}{12}$	$\frac{5\pi}{6}$	$\frac{1}{2}$	1
$\frac{\pi}{2}$	π	0	0
$\frac{7\pi}{12}$	$\frac{7\pi}{6}$	$-\frac{1}{2}$	-1
$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	-1.7
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	-1	-2
$\frac{5\pi}{6}$	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	-1.7
$\frac{11\pi}{12}$	$\frac{11\pi}{6}$	$-\frac{1}{2}$	-1
π	2π	0	0

Continuing this from π to 2π traces out the other two "petals" of this graph, shown in the figure to the right.

(d) $r = 2 + 4\cos\theta$



θ	$\cos\theta$	$4\cos\theta$	$r = 2 + 4\cos\theta$
0	1	4	6
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	3.4	5.4
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	2.8	4.8
$\frac{\pi}{3}$	$\frac{1}{2}$	2	4
$\frac{\pi}{2}$	0	0	2
$\frac{2\pi}{3}$	$-\frac{1}{2}$	-2	0
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	-2.8	-.8
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	-3.4	-1.4
π	-1	-4	-2

Continuing this from π to 2π traces out the other symmetric half of this graph, shown in the figure to the right.